Pion and Kaon Lab Frame Differential Cross Sections for Intermediate Energy Nucleus-Nucleus Collisions

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NASA/TP-2008-215544

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November 2008

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Abstract

Space radiation transport codes require accurate models for hadron production in intermediate energy nucleus - nucleus collisions. Codes require cross sections to be written in terms of lab frame variables and it is important to be able to verify models against experimental data in the lab frame. Several models are compared to lab frame data. It is found that models based on algebraic parameterizations are unable to describe intermediate energy differential cross section data. However, simple thermal model parameterizations, when appropriately transformed from the center of momentum to the lab frame, are able to account for the data.

1 Introduction

Many radiation transport codes are used for transport of nucleons and other hadrons, and are suitable for simulating such things as particle physics experiments and cosmic ray proton propagation through interstellar space and the Earth atmosphere. These codes are also used in proton and hadron therapy applications. Some codes can transport heavy ions and can be used in space radiation applications as well as heavy ion beam therapy. An example is the code HZETRN [1, 2, 3] which is a heavy ion transport code widely used in space applications. Heavy ion transport codes need many atomic, nuclear, and particle physics cross sections as input. The atomic cross sections are needed to describe slowing down of the incoming projectile. A large range of nuclear interactions cross sections including fragmentation, breakup, electromagnetic dissociation [4], and even pair production [5] are needed as input.

A special difficulty of space radiation transport codes is the intermediate energy region in which they must be accurate. Low energy cross sections can be dealt with using non - relativistic formalisms and very high energy cross sections can also be dealt with using ultrarelativistic approximations. However, the peak of the cosmic ray spectrum [6] occurs in the intermediate energy range of several GeV. From a theoretical view, this is a difficult area because non - relativisitic theory is not applicable and ultrarelativistic approximations do not work. Eyser and Machleidt [7] have emphasized this with regard to nucleon - nucleon scattering. Heavy ion transport codes also need to include hadron production from nuclear collisions, which is the subject of the present paper.

Another difficulty faced by many transport codes, including heavy ion space radiation codes, is that cross sections must be calculated in the lab frame because the Boltzmann transport equation is usually set up in such a way that projectiles are transported through a stationary target. Particle physics cross sections, however, are much easier to calculate in the nucleon - nucleon center of momentum (cm) frame. Although the techniques for Lorentz transforming to the lab frame are well known, one must deal with extra

complications such as double valued functions of angles [8] and the fact that cross sections in the lab frame are much more complicated than their relatively simple counterparts in the cm frame. In summary, some of the difficulties faced by heavy ion transport codes for space radiation applications are:

1) They must include all relevant atomic, nuclear and particle physics effects.

2) They must be accurate in the theoretically difficult intermediate energy range.

3) All cross sections must be transformed to the lab frame.

The aim of the present paper is to discuss parameterizations of pion and kaon production in heavy ion collisions at intermediate energy in the lab frame. These parameterizations will be suitable for use in heavy ion transport codes for space radiation applications.

2   Differential cross sections

The Lorentz - invariant differential cross sections of Badhwar [9] and the thermal model have very simple forms and they are *both* expressed in the nucleon - nucleon center of momentum frame. Even though they are Lorentz - invariant, we shall always want to plot them as functions of non - invariant variables and indeed they are written in terms of non - invariant variables. The Nagamiya data [10] is given in the lab frame and so we must transform the simple looking Badhwar and thermal model cross sections into complicated lab functions. Furthermore, transport codes require all cross sections in the lab frame. The techniques for performing the Lorentz transformations are described below.

2.1   Review of kinematics

The appropriate relativistic kinematics is reviewed in this section, so that one can understand the quantities used in the algebraic parameterizations, which are introduced later. Consider the *inclusive* reaction

$$a + b \rightarrow c + X \ , \tag{1}$$

where c is the produced particle of interest and X is anything. Throughout this paper we assume that all variables, such as all momenta, are evaluated in the center of momentum (cm) frame, unless otherwise indicated. Lab frame variables will be given a subscript. For example, the variable x evaluated in the cm frame is written as x but evaluated in the lab frame, it is written as x_{lab}. The momentum of particle c is denoted as p, and supposing that it scatters at angle θ to the beam direction, then the longitudinal and transverse components of momentum are

$$p_z \quad \equiv \quad p \cos \theta \ , \tag{2}$$
$$p_T \quad \equiv \quad p \sin \theta \ . \tag{3}$$

Note that

$$p^2 = p_z^2 + p_T^2 , \tag{4}$$
$$\tan\theta = p_T/p_z . \tag{5}$$

Feynman used a scaled variable instead of p_z itself [11, 12, 13, 14]. The *Feynman scaling variable* is [13, 15, 16, 17, 18, 19, 20]

$$x_F \equiv \frac{p_z}{p_{z\,\text{max}}} , \tag{6}$$

where p_z is the longitudinal momentum of the produced meson in the cm frame, and $p_{z\,\text{max}}$ is the maximum transferable momentum given by [16, 17, 20]

$$p_{z\,\text{max}} = \sqrt{\frac{\lambda(s, m_c, m_X)}{4s}} , \tag{7}$$

with

$$\lambda(s, m_i, m_j) \equiv (s - m_i^2 - m_j^2)^2 - 4m_i^2 m_j^2 . \tag{8}$$

Here, the Mandelstam variable s is the square root of the center of momentum total energy, and m is the mass of a particle. Note that

$$p_{z\,\text{max}} = p_{\text{max}} , \tag{9}$$

because the component of any vector can be no larger than the magnitude of the vector itself. Nagamiya and Gyulassy [17] point out that if c is a boson with zero baryon number, then

$$m_X = m_A + m_B , \tag{10}$$

in agreement with the $p_{z\,\text{max}}$ formulas of Nagamiya and Gyulassy [17] and Cassing [20]. The Feynman scaling variable approaches the limiting value [18]

$$x_F \to \frac{2p_z}{\sqrt{s}} , \qquad \text{as} \qquad s \to \infty \tag{11}$$

Also, it is obviously bounded in the following manner [13]

$$-1 < x_F < 1 . \tag{12}$$

Sets of variables that are often used are either (p, θ) or (p_z, p_T). Writing

$$p_z = x_F \sqrt{\frac{\lambda(s, m_c, m_X)}{4s}} , \tag{13}$$

3

shows that another useful variable set is (x_F, p_T), which is used by Alt *et al.* [21, 22] when presenting their data. These variables are also used throughout the present work.

Rapidity is defined as

$$y \equiv \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right) \,, \tag{14}$$

so that

$$E = m_T \cosh y \,, \tag{15}$$
$$p_z = m_T \sinh y \,, \tag{16}$$

where the transverse mass is defined through

$$m_T^2 \equiv m^2 + p_T^2 = E^2 - p_z^2 \,, \tag{17}$$

with m is the mass of the produced particle c. This gives yet another useful variable set (y, p_T). In the work below, it will be necessary to write the rapidity in terms of the Feynman scaling variable as

$$y = \frac{1}{2} \log \left(\frac{\sqrt{x_F^2 + m_T^2/p_{z\,\text{max}}^2} + x_F}{\sqrt{x_F^2 + m_T^2/p_{z\,\text{max}}^2} - x_F} \right) \,. \tag{18}$$

2.2 Lorentz transformations

The relativistic gamma factor γ_p for the projectile nucleus in the lab frame is given by

$$\gamma_p = T_p/m + 1 \,, \tag{19}$$

where T_p is the kinetic energy of the projectile nucleus in units of *energy/nucleon* and m is the mass of the nucleon. Using

$$\gamma_p \equiv \frac{1}{\sqrt{1 - \beta_p^2}} \,, \tag{20}$$

gives the speed β_p of the nucleus in the lab frame. This is also the gamma factor for a particular *nucleon* in the lab frame and β_p is the speed of a particular nucleon in the lab frame.

Denote the *nucleon - nucleon* center of momentum frame as NNcm and the *nucleus - nucleus* center of momentum frame as AAcm. We want the speed of the NNcm in the lab frame, which we denote simply as β. The relativistic gamma factor for the NNcm in the lab frame is

$$\gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} \,. \tag{21}$$

The *relation* between the two gamma factors is given by

$$\gamma_p = 2\gamma^2 - 1 . \tag{22}$$

This relates the speed of the NNcm in the lab frame to the speed of a projectile nucleon in the lab frame. The gamma factor that appears in the Lorentz transformations below is

$$\gamma = \sqrt{\frac{1}{2}(\gamma_p + 1)} . \tag{23}$$

This γ enables us to transform quantities from the NNcm frame to the lab frame. One can now see how this is obtained from the nucleus kinetic energy T_p in equation (19).

The cm frame moves at speed β relative to the lab. The Lorentz transformations are

$$\left(\begin{array}{c} E \\ p_z \end{array} \right) = \left(\begin{array}{cc} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{array} \right) \left(\begin{array}{c} E_{\text{lab}} \\ p_{z\,\text{lab}} \end{array} \right) , \tag{24}$$

with the transverse momentum p_T being the same in both frames. The parameterizations below are often written in terms of the cm variables (p_z, p_T). The explicit Lorentz transformation to $(p_{z\,\text{lab}}, p_{T\,\text{lab}})$ variables is given by

$$p_z = -\gamma\beta\sqrt{p_{z\,\text{lab}}^2 + p_{T\,\text{lab}}^2 + m^2} + \gamma p_{z\,\text{lab}} , \tag{25}$$
$$p_T = p_{T\,\text{lab}} . \tag{26}$$

Instead of $(p_{z\,\text{lab}}, p_{T\,\text{lab}})$ variables, we will also need to use $(p_{\text{lab}}, \theta_{\text{lab}})$ variables. The explicit transformation to the momentum and angle lab variables is

$$p_z = -\gamma\beta\sqrt{p_{\text{lab}}^2 + m^2} + \gamma p_{\text{lab}} \cos\theta_{\text{lab}} , \tag{27}$$
$$p_T = p_{\text{lab}} \sin\theta_{\text{lab}} . \tag{28}$$

Simple substitution enables one to write the rapidity and Feynman scaling variable in terms of lab variables.

2.3 Algebraic parameterizations

Badhwar and Stephens [9, 23] have provided parameterizations of inclusive pion and kaon production from proton - proton collisions. These parameterizations are for high energy and are given in the *nucleon - nucleon* center of momentum frame (NNcm), and they are given for both charged and neutral pions and for charged kaons. These parameterizations have been transformed to the lab frame using the techniques above. The results for charged and neutral pions are plotted in figure 1. As expected, the cross sections for π^+ are the largest, π^- are the smallest and π^0 are in between. The charged kaon cross sections are plotted in figure 2. It can be seen that the parameterizations are much smaller than the pion cross sections and again the K^+ cross sections are bigger than the K^- cross sections. Other parameterizations have also been developed by Alper [24], Mokhov [25], Ellis [26] and Carey [27]. Details of the various parameterizations are now given.

2.3.1 Badhwar parameterization

The Badhwar parameterization [9] gives the Lorentz - invariant differential cross section for charged pions as

$$E\frac{d^3\sigma}{d^3p}(\pi^\pm) = \frac{A}{(1+4m_p^2/s)^r}(1-\tilde{x})^q\exp[\frac{-Bp_T}{1+4m_p^2/s}]\,,\tag{29}$$

and neutral pions as

$$E\frac{d^3\sigma}{d^3p}(\pi^0) = Af(E_p)(1-\tilde{x})^q\exp[\frac{-Bp_T}{1+4m_p^2/s}]\,,\tag{30}$$

and charged kaons as

$$E\frac{d^3\sigma}{d^3p}(K^\pm) = A(1-\tilde{x})^C\exp(-Bp_T)\,,\tag{31}$$

where m_p is the proton mass, $\sqrt{s}$ is the total energy in the center of momentum (cm) frame, and p_T is the transverse momentum of the produced meson in the cm frame. The other terms are given by

$$\tilde{x} = \left[x_F^2 + \frac{4}{s}(p_T^2+m^2)\right]^{1/2}\,,\tag{32}$$

where it is assumed that the variables appearing in x_F are in the cm frame. The mass m is the mass of the produced particle (pion or kaon). Badhwar writes $x_\parallel^* \equiv x_F$. Also,

$$q = \frac{C_1+C_2p_T+C_3p_T^2}{\sqrt{1+4m_p^2/s}}\,.\tag{33}$$

The function $f(E_p)$ for neutral pions is given by

$$f(E_p) = (1+23E_p^{-2.6})(1-4m_p^2/s)^r\,,\tag{34}$$

with the constants listed in Table 1. Badhwar points out that for large values of E_p, equation (30) takes the asymptotic form

$$E\frac{d^3\sigma}{d^3p}(\pi^0) = A\exp(-Bp_T)(1-\tilde{x})^{(C_1-C_2p_T+C_3p_T^2)}\,,\tag{35}$$

which is consistent with the Feynman scaling hypothesis [13]. The Badhwar variables are (x_F, p_T), which are also used in the Alt *et al.* [21, 22] data, so that no variable conversion is necessary.

Table 1: Constants for the Badhwar parameterization.

Particle	A	B	r	C	C_1	C_2	C_3
π^+	153	5.55	1	$\cdots$	5.3667	- 3.5	0.8334
π^-	127	5.3	3	$\cdots$	7.0334	- 4.5	1.667
π^0	140	5.43	2	$\cdots$	6.1	3.3	0.6
K^+	8.85	4.05	$\cdots$	2.5	$\cdots$	$\cdots$	$\cdots$
K^-	9.3	3.8	$\cdots$	8.3	$\cdots$	$\cdots$	$\cdots$

2.3.2 Alper parameterization

The Alper [24] parameterization for charged pions and kaons, protons and antiprotons is

$$E\frac{d^3\sigma}{d^3p} = A_1 \exp(-Bp_T)\exp(-Dy^2) + A_2\frac{(1 - p_T/p_{\text{beam}})^m}{(p_T^2 + M^2)^n} \; , \tag{36}$$

with the constants listed in Table 2. The Alper variables are (y, p_T). To change to the variables (x_F, p_T), we convert the rapidity in equation (36) to x_F using equation (18).

Table 2: Constants for the Alper parameterization.

Particle	A_1	B	D	A_2	M	m	n
π^+	210	7.58	0.20	10.7	1.03	10.9	4.0
π^-	205	7.44	0.21	12.8	1.08	13.1	4.0
K^+	14.3	6.78	1.5	8.0	1.29	12.1	4.0
K^-	13.4	6.51	1.8	9.8	1.39	17.4	4.0
p	5.3	3.8	- 0.2	16	1.2	0	7.5
$\bar{p}$	1.89	4.1	2.3	25	1.41	25	4.5

2.3.3 Ellis parameterization

The Ellis [26] parameterization for charged pions, neutral pions, charged kaons, protons and antiprotons is

$$E\frac{d^3\sigma}{d^3p} = A(p_T^2 + M^2)^{-N/2}(1 - x_T)^F \; , \tag{37}$$

where A is an overall normalization fitted to be $A = 13$ in reference [28] and $x_T \equiv p_T/p_{\max} \approx 2p_T/\sqrt{s}$. The same value of A is used in the present work. The other constants are listed in Table 3. The Ellis parameterization is independent of the emission angle θ, and so does not carry any dependence on p_z, x_F, y etc.

Table 3: Constants for the Ellis parameterization.

Particle	N	M^2	F
π^+	7.70	0.74	11.0
π^-	7.78	0.79	11.9
π^0	10.8	2.3	7.1
K^+	8.72	1.69	9.0
K^-	8.76	1.77	12.2
p	10.38	1.82	7.3
$\bar{p}$	9.1	1.17	14.0

2.3.4 Mokhov parameterization

The Mokhov [25] parameterization is

$$E\frac{d^3\sigma}{d^3p} = A\left(1 - \frac{p}{p_{\max}}\right)^B \exp\left(-\frac{p}{C\sqrt{s}}\right) V_1(p_T)V_2(p_T) , \tag{38}$$

where

$$
\begin{aligned}
V_1(p_T) &= (1-D)\exp(-Ep_T^2) + D\exp(-Fp_T^2) && \text{for } p_T \le 0.933 \text{ GeV,} \\
V_1(p_T) &= \frac{0.2625}{(p_T^2 + 0.87)^4} && \text{for } p_T > 0.933 \text{ GeV,}
\end{aligned} \tag{39}
$$

and

$$
\begin{aligned}
V_2(p_T) &= 0.7363\exp(0.875p_T) && \text{for } p_T \le 0.35 \text{ GeV,} \\
V_2(p_T) &= 1 && \text{for } p_T > 0.35 \text{ GeV,}
\end{aligned} \tag{40}
$$

with the constants listed in Table 4. Using $p = \sqrt{p_z^2 + p_T^2}$, gives the Mokhov variables (p_z, p_T) which are transformed to (x_F, p_T) using equation (13).

Table 4: Constants for the Mokhov parameterization.

Particle	A	B	C	D	E	F
π^+	60.1	1.9	0.18	0.3	12	2.7
π^-	51.2	2.6	0.17	0.3	12	2.7

2.3.5 Carey parameterization

The Carey [27] parameterization, for negative pions, negative kaons, and antiprotons is

$$E\frac{d^3\sigma}{d^3p} \;=\; hN(p_T^2 + G)^{-4.5}(1 - x_R)^J \;, \tag{41}$$

where N is an overall normalization fitted to be $N = 13$ in reference [28] and $x_R \equiv p/p_{\max} \approx 2p/\sqrt{s}$. The same value of N is used in the present work. The constants are listed in Table 5. The Carey variables are (p_z, p_T). To change to the variables (x_F, p_T), we use $x_R = \sqrt{x_F^2 + p_T^2/p_{\max}^2}$.

Table 5: Constants for the Carey parameterization.

Particle	N	h	G	J
π^-	13	1.0	0.86	4
K^-	13	0.36	1.22	5
$\bar{p}$	13	0.26	1.04	7

2.4 Thermal model

Several authors [10, 17, 29, 30] have shown that a simple thermal model parameterization is able to describe both pion and kaon data in intermediate energy nucleus - nucleus collisions. In the *nucleon - nucleon* cm frame, this parameterization is given by

$$E\frac{d^3\sigma}{d^3p} = N\exp(-T/E_0) \;, \tag{42}$$

where T is the meson kinetic energy in the *nucleon - nucleon* cm frame. (It is *not* the nucleus - nucleus cm frame.) The parameters N and E_0 are constants fitted to the data.

These parameters are different for protons, pions and kaons. Some effective nuclear temperatures E_0 are listed in Table 6, which have been taken from references [10, 17, 30]. We use the wording "effective" nuclear temperature instead of just nuclear temperature because there is not a single well defined temperature. It is different for different particles, such as pions and kaons. Equation (42) is characteristic of a thermal Boltzmann distribution, and describes the "boiling" off of hadrons after nuclear equilibration has been reached. Clearly, this model is vastly different from a nucleon - nucleon collision model assumed above.

Table 6: Effective nuclear Boltzmann temperatures E_0.

Initial State	Projectile Energy (A GeV)	Produced Particle	E_0 (MeV)	References
Ar + Pb	0.8	π^-	72	figure 19 [10]
Ar + Pb	0.8	p	87	figure 19 [10]
Ar + KCl	0.8	π^-	66	figure 19 [10]
Ar + KCl	0.8	p	79	figure 19 [10], figure 4 [30]
C + C	0.8	π^-	60	figure 19 [10]
C + C	2.1	π^-	123	figure 27 [17]
C + C	0.8	p	68	figure 19 [10], figure 4 [30]
Ne + NaF	0.18	π^-	35	figure 27, 28 [17]
Ne + NaF	0.4	π^-	39	figure 28 [17]
Ne + NaF	0.8	π^-	62	figure 19 [10], figure 28 [17]
Ne + NaF	2.1	π^-	102	figure 28 [17]
Ne + NaF	0.8	p	75	figure 19 [10], figure 4 [30]
Ne + NaF	2.1	K^+	142	figure 35 [17], figure 5 [30] See also [29]

3 Comparison to experiment

Experimental data for π^- production in the Ar + KCl nuclear collisions at 800 A MeV have been measured by Nagamiya *et al.* [10]. They measured Lorentz - invariant differential cross sections as a function of transverse momentum and lab angle.

The nuclear radius is given by the well known relation $R \approx 1.2$ fm $A^{1/3}$, and therefore a total cross section for a nucleus scales like $A^{2/3}$ multiplied by a nucleon cross section. The nucleus - nucleus geometrical cross section [10, 17, 31] therefore scales as $(A_P^{2/3} + A_T^{2/3})$. However, pion production is more dependent on the number of binary nucleon - nucleon

collisions, which has a different scaling function [10]. The Badhwar parameterization is scaled up to nucleus - nucleus collisions by multiplying the nucleon - nucleon cross section by $(A_P A_T)^{2/3}$, where A_P and A_T is the projectile and target nucleon numbers. As discussed above, this is then transformed to the lab frame. The results are compared to the Nagamiya data in figures 3 and 4. It can be seen that the scaled Badhwar parameterization fails to describe the data. Even if the scaling factor is changed to an arbitrary constant, the agreement with the spectral shapes is still very poor. Extensive work has also been done on the parameterizations of Alper [24], Mokhov [25], Ellis [26] and Carey [27]. These parameterizations have been treated in the same fashion as the Badhwar parameterization and compared to data. All of them also fail to describe the data, with comparisons of similar quality to the Badhwar comparison.

The failure of algebraic parameterizations is not unexpected. They were developed to describe high energy nucleon - nucleon collisions in the ultrarelativistic limit, whereas the Nagamiya data describe intermediate energy nucleus - nucleus collisions.

The pion thermal model parameterization, transformed to the lab frame, is shown in figures 5 and 6. The parameter N is fitted to the data with

$$N = 1.7 \times 10^4 \frac{\text{mb}}{\text{GeV}^2\text{sr}} = 0.017 \frac{\text{mb}}{\text{MeV}^2\text{sr}}$$

The effective nuclear temperature is taken from Table 6 which is $E_0 = 66$ MeV. Comparison to experiment is very good. *It is clear that the rather complicated experimental distributions as a function of lab angle reveal themselves to be nothing more than a simple NNcm thermal distribution appropriately transformed to the lab frame.* Finally, the kaon thermal model parameterization, transformed to the lab frame, is shown in figure 7. The reason this is plotted is for comparison to the pion spectra.

4   Extension to other nuclei, energy and pion species

Nagamiya *et al.* [10] have discussed pion production from arbitrary nuclei at arbitrary energy. This is needed for transport codes such as HZETRN. The comparisons presented so far have been for the Ar + KCl system with projectile mass number of $A_P = 40$ and target mass number $A_T = 37$ where the latter is the average of $A_K = 39$ and $A_{Cl} = 35$. Nagamiya *et al.* [10] show that a *simplified* dependence is

$$E\frac{d^3\sigma}{d^3p} \propto (A_P A_T)^{5/3} \, . \tag{43}$$

Equation (42) is true for $A_P = 40$ and $A_T = 37$. Equation (43) means that equation (42) can be generalized to

$$E\frac{d^3\sigma}{d^3p} = \left(\frac{A_P A_T}{40 \times 37}\right)^{5/3} N \exp(-T/E_0) \, . \tag{44}$$

for *arbitrary* values of A_P and A_T. In figure 25 of reference [10], the mass dependence of the effective nuclear temperature E_0 is shown. This temperature can be parameterized as

$$E_0 = 70\sqrt{\mathrm{T_{lab}}/1000} \, , \tag{45}$$

with $\mathrm{T_{lab}}$ and E_0 both in units of MeV. Thus, the final generalized cross section is

$$E\frac{d^3\sigma}{d^3p} = 0.017 \left(\frac{A_P A_T}{40 \times 37} \right)^{5/3} \exp\left(-\frac{T}{70\sqrt{\mathrm{T_{lab}}/1000}} \right) \, , \tag{46}$$

with all energies in MeV, resulting in the cross section $E\frac{d^3\sigma}{d^3p}$ with units of $\frac{\mathrm{mb}}{\mathrm{MeV^2 sr}}$. *This generalization is expected to be accurate only for energies* $\mathrm{T_{lab}} < 5$ AGeV *and should not be used for higher energy.*

Finally, the cross sections developed in the present work have been applied to π^- production only. From figure 30 of Nagamiya *et al.* [10], one can see that to a *rough approximation*, the ratio of positive to negative pions is roughly unity. Employing the well known relation $\sigma(\pi^0) \approx \frac{1}{2}[\sigma(\pi^+) + \sigma(\pi^-)]$ shows that the cross section for all pion species is the same if the cross sections for positive and negative pions is the same. Thus, in the present work, the approximation that the cross section is the same for all pion species is used. *This approximation should be improved in future work.*

5 Conclusions

Space radiation transport codes require accurate models describing hadron production in intermediate energy nucleus - nucleus collisions. These models must be written in terms of lab frame variables. Several models have been compared to the intermediate energy data of Nagamiya [10], which has been presented in terms of lab frame variables. The first set of models used high energy algebraic parameterizations multiplied by $(A_P A_T)^{2/3}$. (Scaling by $(A_P A_T)^{5/3}$ also fails for these models.) Comparison between theory and experiment was poor. Such models should not be used in 3 - dimensional transport codes. A simple thermal model parameterization was found to satisfactorily describe the data. Because of this success, it is recommended that such thermal model parameterizations should be utilized in 3 - dimensional space radiation transport codes.
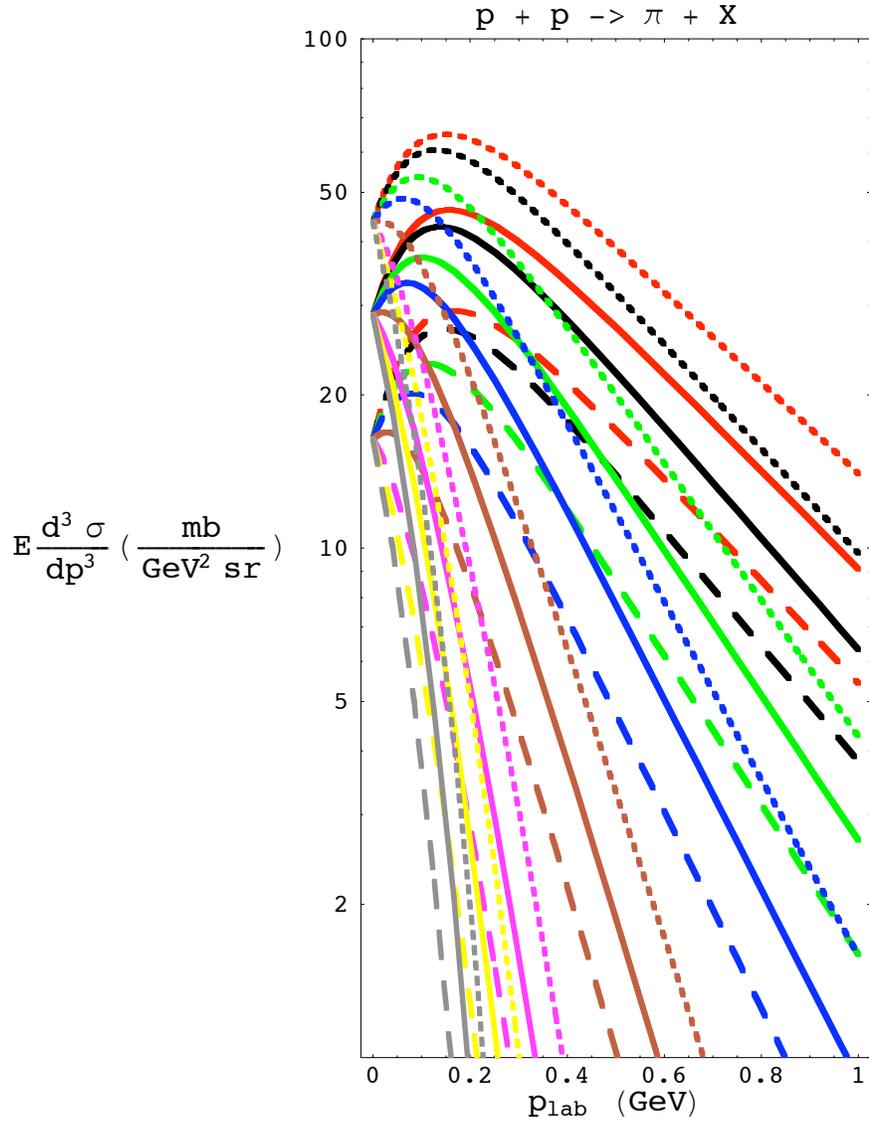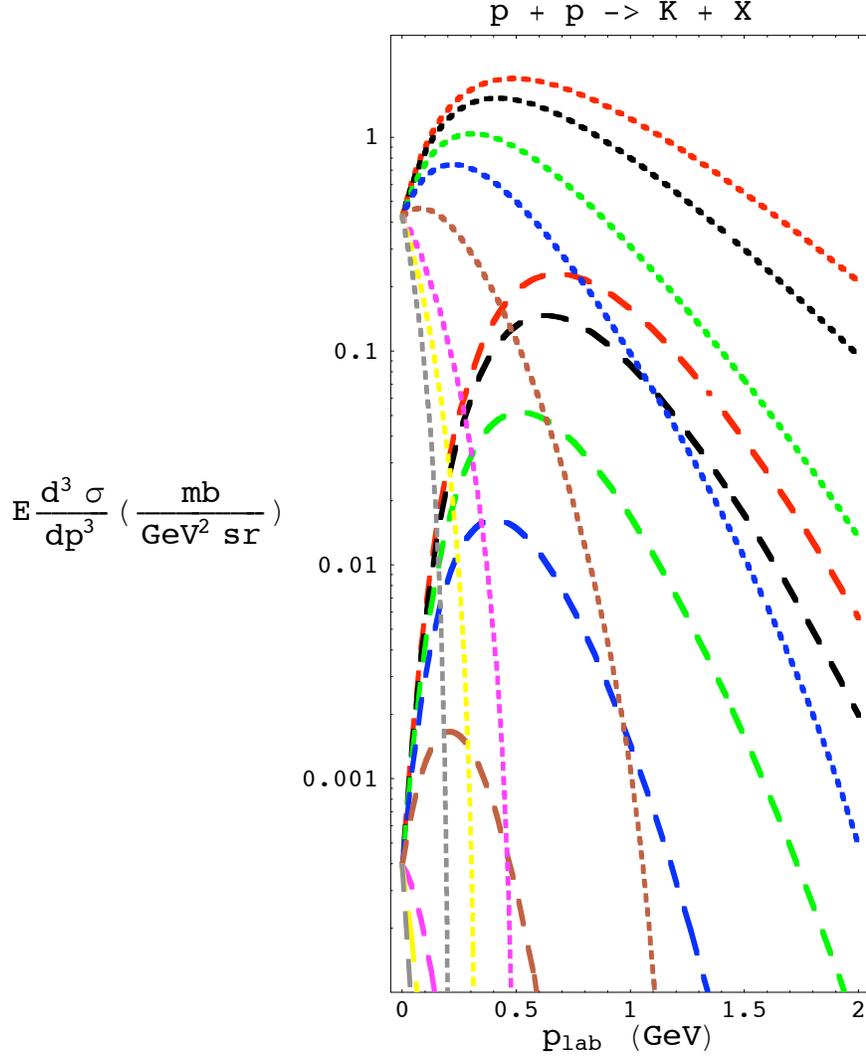
Figure 1: Pion inclusive Lorentz invariant differential cross sections as a function of lab momentum and angle for proton - proton collisions with an incident proton kinetic energy of $T_{\text{lab}} = 5$ GeV. The solid lines are π^0, the long dashed lines are π^- and the short dashed lines are π^+. The lab angles from top to bottom are $15°$, $20°$, $30°$, $40°$, $60°$, $90°$, $110°$ and $145°$. The curves have been generated using the Badhwar and Stephens parameterizations [9, 23] and by transforming to the lab frame.

Figure 2: Kaon inclusive Lorentz invariant differential cross sections as a function of lab momentum and angle for proton - proton collisions with an incident proton kinetic energy of $T_{\text{lab}} = 5$ GeV. The long dashed lines are K^- and the short dashed lines are K^+. The lab angles from top to bottom are $15°$, $20°$, $30°$, $40°$, $60°$, $90°$, $110°$ and $145°$. The curves have been generated using the Badhwar and Stephens parameterizations [9] and by transforming to the lab frame.

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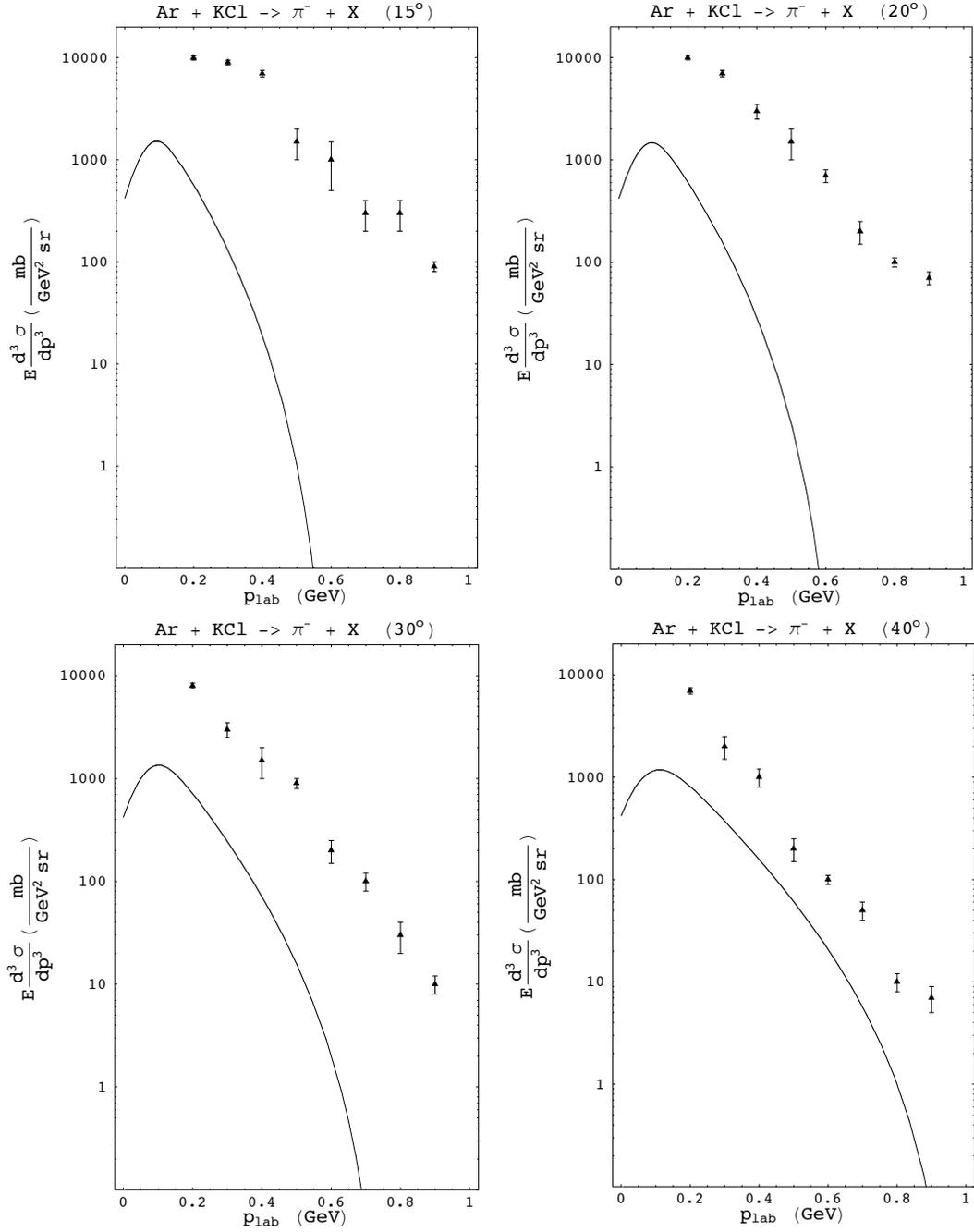
Figure 3: Badhwar parameterization [9] multiplied by $(A_P A_T)^{2/3}$ fails to describe the data [10] at 800 A MeV. Lab angles are indicated. (Parameterizations of Alper [24], Carey [27], Ellis [26] and Mokhov [25] are of similar poor quality.)

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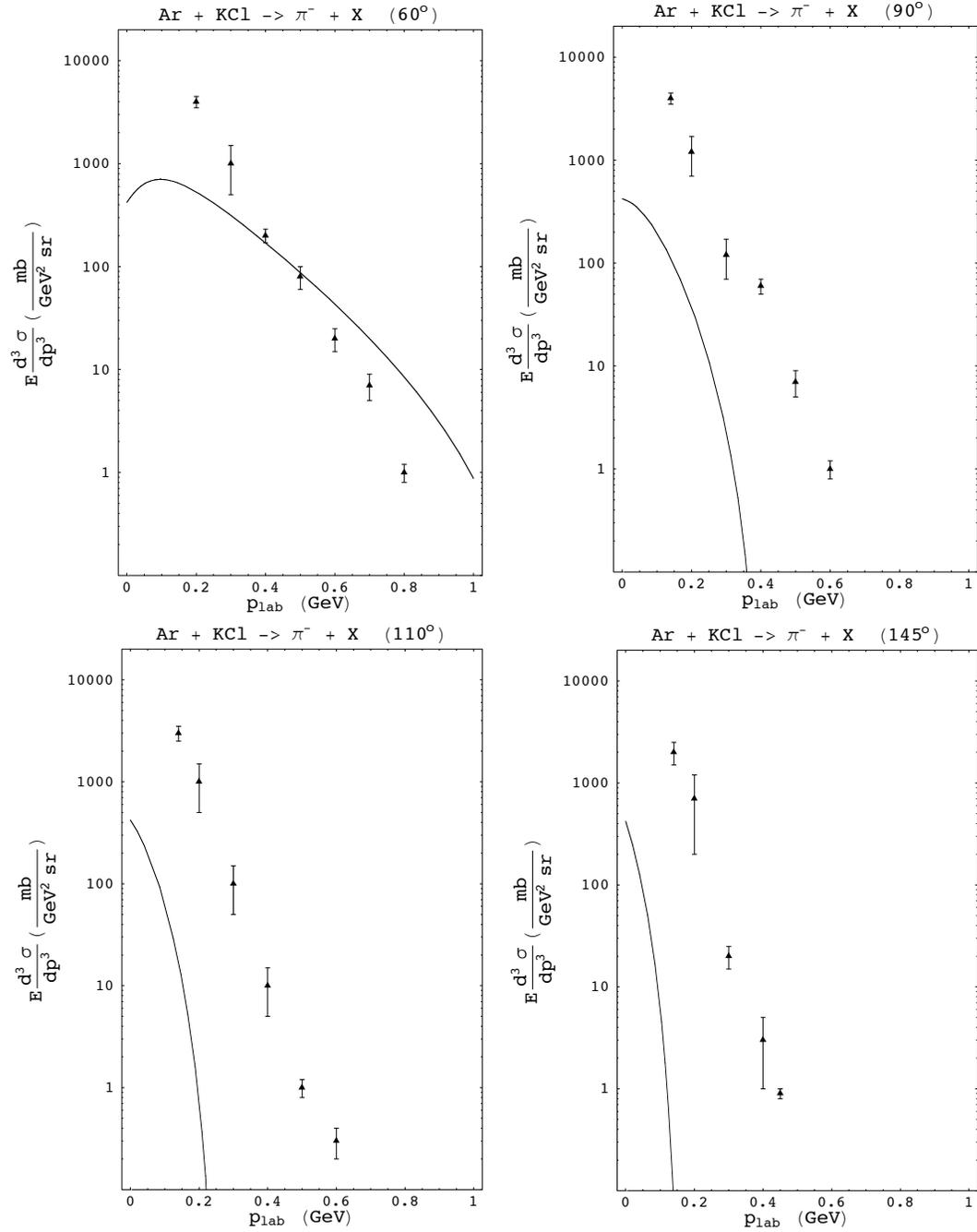
Figure 4: Same as figure 3, except different lab angles. (Parameterizations of Alper [24], Carey [27], Ellis [26] and Mokhov [25] are of similar poor quality.)
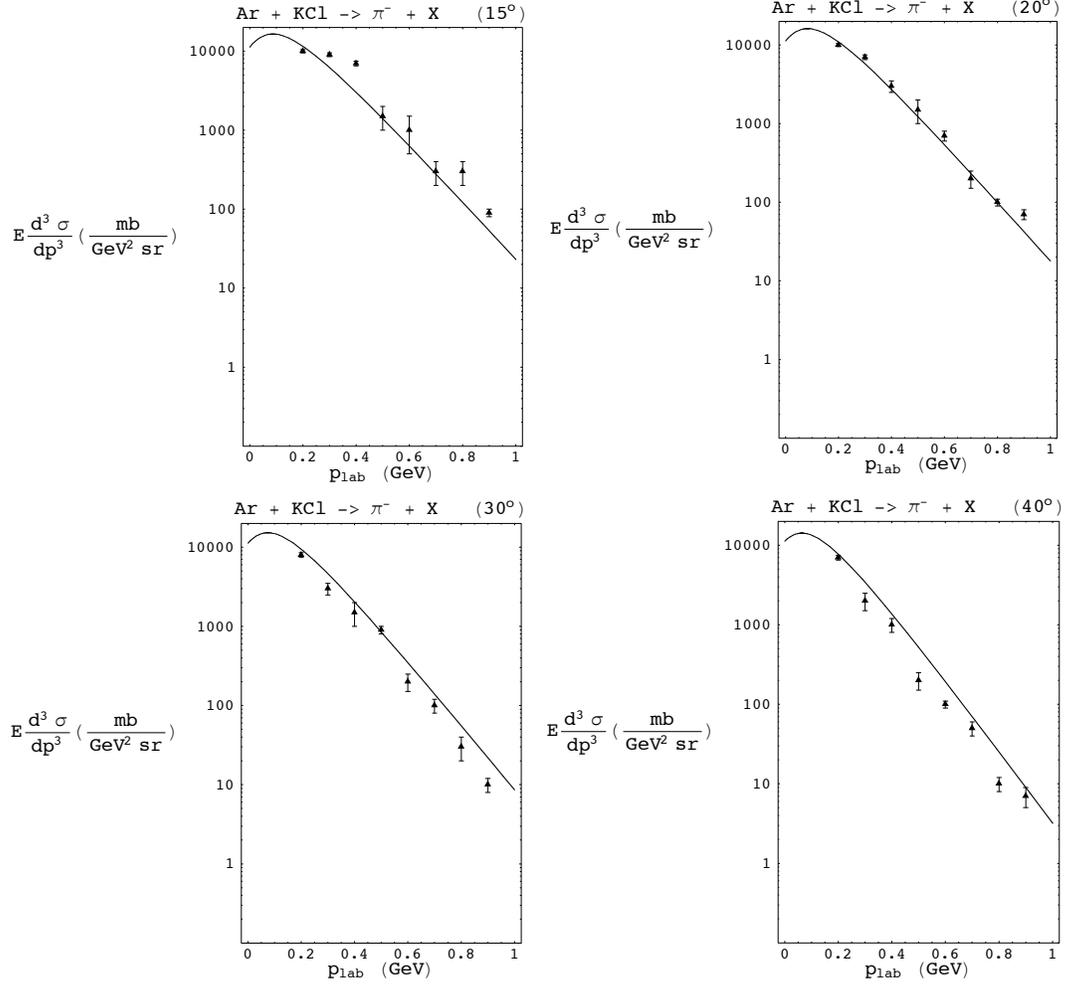
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Figure 5: Thermal spectrum successfully describes data [10] at 800 A MeV. The overall normalization from equation (44) was fitted as $N = 1.7 \times 10^4$ mb/GeV^{2}sr. The effective nuclear temperature, taken from reference [10], is $E_0 = 66$ MeV. Lab angles are indicated.
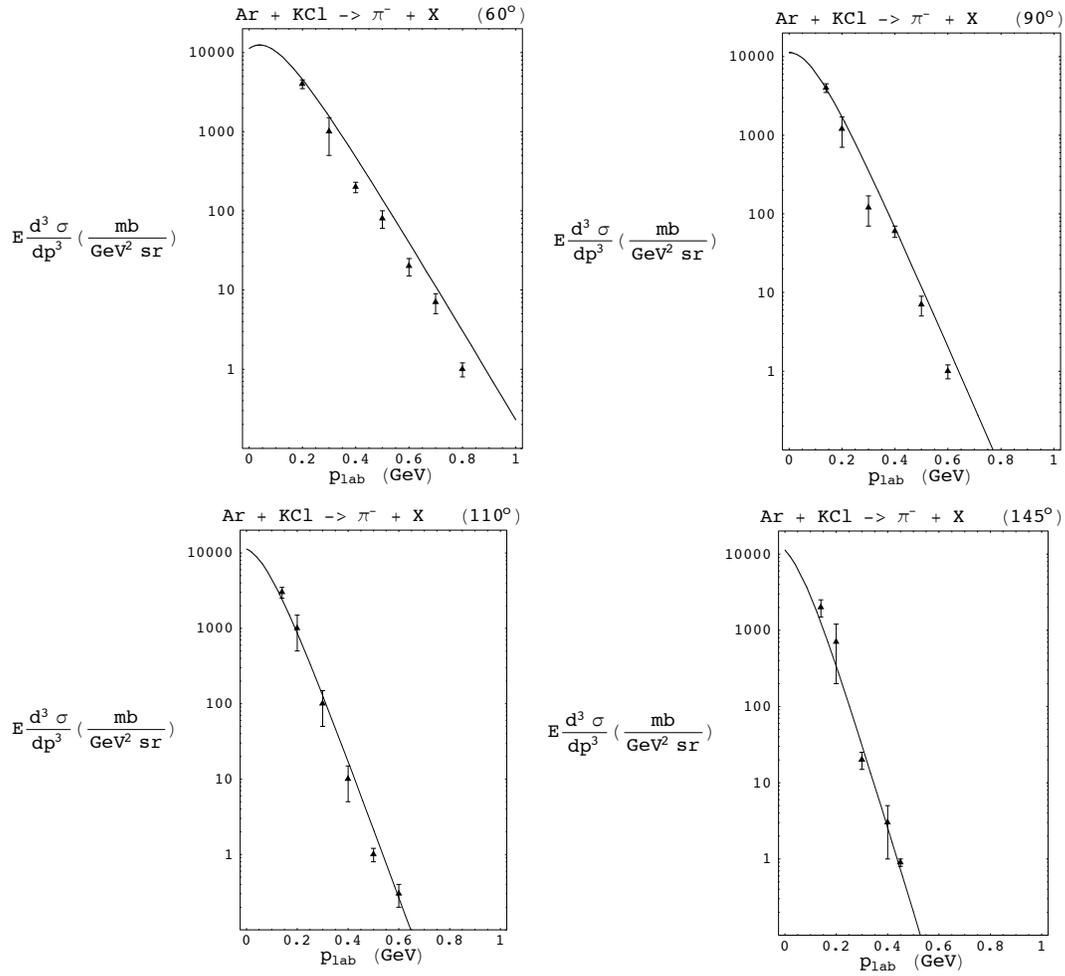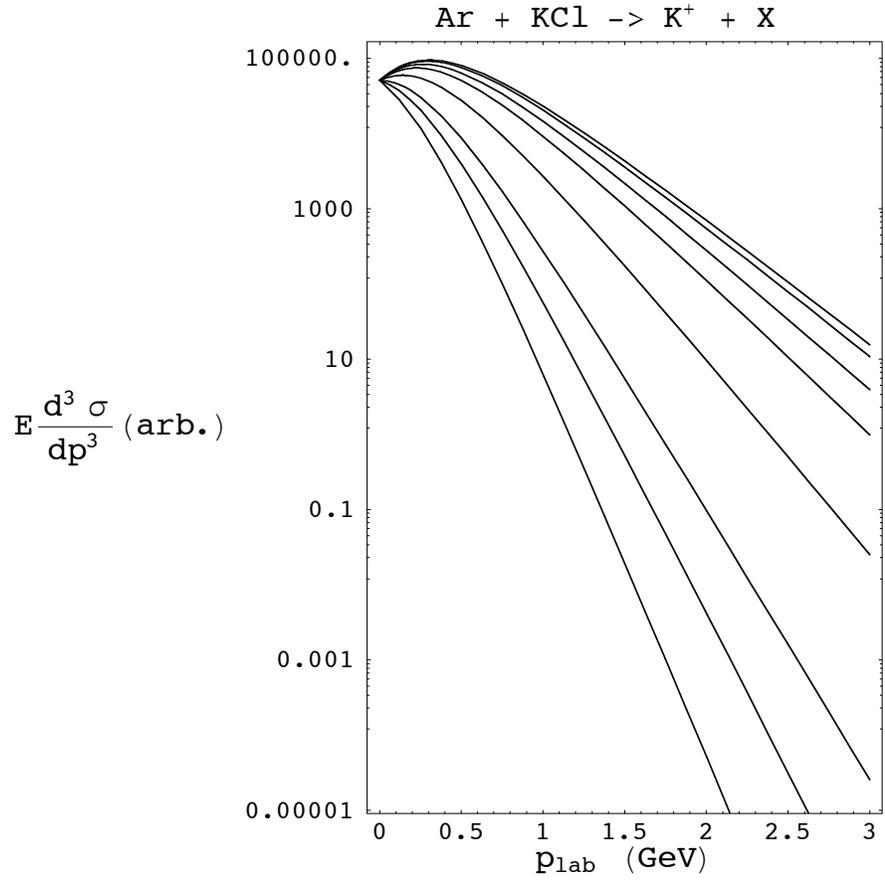
Figure 6: Same as figure 5, except different lab angles.

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Figure 7: K^+ spectra at 2.1 A GeV, with the effective nuclear temperature, taken from references [17, 30], is $E_0 = 142$ MeV. Overall normalization is arbitrary. Lab angles from top to bottom are 15°, 20°, 30°, 40°, 60°, 90°, 110°, 145°.

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REPORT DOCUMENTATION PAGE

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
01-11-2008	Technical Publication	

4. TITLE AND SUBTITLE

Pion and Kaon Lab Frame Differential Cross Sections for Intermediate Energy Nucleus-Nucleus Collisions

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Norbury, John W.; and Blattnig, Steve R.

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

651549.02.07.01

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

NASA Langley Research Center
Hampton, VA 23681-2199

8. PERFORMING ORGANIZATION REPORT NUMBER

L-19385

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSOR/MONITOR'S ACRONYM(S)

NASA

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

NASA/TP-2008-215544

12. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified - Unlimited
Subject Category 93
Availability: NASA CASI (301) 621-0390

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Space radiation transport codes require accurate models for hadron production in intermediate energy nucleus-nucleus collisions. Codes require cross sections to be written in terms of lab frame variables and it is important to be able to verify models against experimental data in the lab frame. Several models are compared to lab frame data. It is found that models based on algebraic parameterizations are unable to describe intermediate energy differential cross section data. However, simple thermal model parameterizations, when appropriately transformed from the center of momentum to the lab frame, are able to account for the data.

15. SUBJECT TERMS

Meson production; Space radiation

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			STI Help Desk (email: help@sti.nasa.gov)
U	U	U	UU	28	19b. TELEPHONE NUMBER *(Include area code)* (301) 621-0390