Nucleon-Nucleon Total Cross Section

John W. Norbury
Langley Research Center, Hampton, Virginia

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.

- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.

- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results ... even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at *http://www.sti.nasa.gov*

- E-mail your question via the Internet to help@sti.nasa.gov

- Fax your question to the NASA STI Help Desk at (301) 621-0134

- Phone the NASA STI Help Desk at (301) 621-0390

- Write to:
  NASA STI Help Desk
  NASA Center for AeroSpace Information
  7115 Standard Drive
  Hanover, MD 21076-1320

Nucleon-Nucleon Total Cross Section

John W. Norbury
Langley Research Center, Hampton, Virginia

Acknowledgments

This work was supported, in part, by NASA Research grant NNL07AA00C.

Nomenclature

Throughout this paper $\hbar \equiv c \equiv 1$.

$m = 938$ MeV is the nucleon mass, which is used for both protons and neutrons.

E_{1l} is the total energy of the projectile (particle 1) in the lab frame.

T_{1l} is the kinetic energy of the projectile (particle 1) in the lab frame.

p_{1l} is the momentum of the projectile (particle 1) in the lab frame.

10^3 GeV = TeV = 10^{12} eV (Tera-eV).

10^6 GeV = PeV = 10^{15} eV (Peta-eV).

10^9 GeV = EeV = 10^{18} eV (Exa-eV).

10^{12} GeV = ZeV = 10^{21} eV (Zetta-eV).

Contents

List of Figures

List of Tables

Abstract

The total proton-proton and neutron-proton cross sections currently used in the transport codes HZETRN and BRYNTRN show some disagreement with experiment in the GeV and EeV energy ranges. The GeV range is near the region of maximum cosmic ray intensity. It is useful to correct these cross sections, so that predictions of space radiation environments will be more accurate. Parameterizations of nucleon-nucleon total cross sections are developed which are accurate over the entire energy range of the cosmic ray spectrum.

1 Introduction

A current assignment is to improve the elastic neutron differential cross sections currently used in HZETRN [1] and BRYNTRN [2, 3]. These cross sections and the relevant theory are described in Chapter 4 of Reference [4], where it is shown that nucleon-nucleon total cross sections are fundamental inputs to the formulas for the neutron elastic differential cross sections. It was therefore decided to check how well the nucleon-nucleon total cross sections currently used in HZETRN compare with experiment. It will be seen below that although the global fits are quite good, there are some discrepancies in the region of maximum intensity in the cosmic ray spectrum. The aim of the present paper is to make improvements.

Note that the cross sections discussed in this work have also been parameterized by Kalinovskii *et al* [5].

The total (elastic plus inelastic) cross sections for proton-proton (pp), neutron-neutron (nn) and neutron-proton (np) scattering have been parametrized in reference [4]. The pp cross section is

$$\sigma_{pp}^{HZETRN}(T_{1l}) = \left(1 + \frac{5}{T_{1l}}\right)\left\{40 + 109\cos(0.199\sqrt{T_{1l}})\exp\left[-0.451(T_{1l} - 25)^{0.258}\right]\right\}$$
$$\text{(for } T_{1l} \geq 25 \text{ MeV)} \quad (1)$$

$$\sigma_{pp}^{HZETRN}(T_{1l}) = \exp\left\{6.51\left[\exp\left(-\left(\frac{T_{1l}}{134}\right)^{0.7}\right)\right]\right\} \quad \text{(for } T_{1l} < 25 \text{ MeV)} \quad (2)$$

Note that this second equation is written with a typographical error in reference [4], with the minus sign in the wrong place. Note however that this typographical error is not the cause of the discrepancies noted in the present work. The actual formulas currently used in the transport code HZETRN [1] do not have the typographical error and are written correctly, but nevertheless show some disagreement with experiment as explained below.

The total cross section for neutron-neutron scattering is not available and therefore

7

the proton-proton cross section is used, namely

$$\sigma_{nn}(T_{1l}) = \sigma_{pp}(T_{1l}) \tag{3}$$

This is accurate due to isospin symmetry. The neutron-proton cross section is

$$\sigma_{np}^{HZETRN}(T_{1l}) = 38 + 12500 \exp\left[-1.187(T_{1l} - 0.1)^{0.35}\right] \qquad \text{(for } T_{1l} \geq 0.1 \text{ MeV)} \tag{4}$$

$$\sigma_{np}^{HZETRN}(T_{1l}) = 26000 \exp\left[-\left(\frac{T_{1l}}{0.282}\right)^{0.3}\right] \qquad \text{(for } T_{1l} < 0.1 \text{ MeV)} \tag{5}$$

Again, this second equation is written with a typographical error in reference [4], with the minus sign in the wrong place. In the above equations, T_{1l} is the projectile *kinetic energy* as measured in the lab (target) frame in units of MeV. The cross sections are in units of mb.

Many parametrizations found in the literature, as well as experimental data [6], are in units of GeV and therefore, when comparing to data and other work it is more convenient to use GeV. The formulas above take on a different form when expressed in GeV. In order to do this note the following. Consider equation (1). The terms $5/T_{1l}$ and the arguments of cos and exp are obviously dimensionless. This means for example, that the 5 should really read 5 MeV, with the same units as T_{1l}. Thus, in GeV this must be written as $5000/T_{1l}$. Also the number 0.199 must have units of energy$^{-1/2}$. The number 25 must have units of energy, and in GeV units it becomes 25,000 GeV. The number 0.451 must have units of energy$^{-0.258}$, so that it becomes $0.451/1000^{0.258}$ GeV$^{-0.258}$.

The formulas are conveniently re-written as

$$\sigma_{pp}^{HZETRN}(T_{1l}) = \left(1 + \frac{a}{T_{1l}}\right)\left\{40 + 109 \cos(b\sqrt{T_{1l}}) \exp\left[-c(T_{1l} - d)^{0.258}\right]\right\}$$
$$\text{(for } T_{1l} \geq 25 \text{ MeV)} \tag{6}$$

$$\sigma_{pp}^{HZETRN}(T_{1l}) = \exp\left\{6.51 \left[\exp\left(-\left(\frac{T_{1l}}{e}\right)^{0.7}\right)\right]\right\} \qquad \text{(for } T_{1l} < 25 \text{ MeV)} \tag{7}$$

$$\sigma_{np}^{HZETRN}(T_{1l}) = 38 + 12500 \exp\left[-f(T_{1l} - g)^{0.35}\right] \qquad \text{(for } T_{1l} \geq 0.1 \text{ MeV)} \tag{8}$$

$$\sigma_{np}^{HZETRN}(T_{1l}) = 26000 \exp\left[-\left(\frac{T_{1l}}{h}\right)^{0.3}\right] \qquad \text{(for } T_{1l} < 0.1 \text{ MeV)} \tag{9}$$

where the constants a, b, c, d, e, f, g and h are given in Table 1 in the MeV unit system. It then becomes a straightforward matter to convert to GeV.

Table 1: Constants for Cross Section Formulas (6) - (9).

a	b	c	d
5 MeV	0.199 MeV$^{-1/2}$	0.451 MeV$^{-0.258}$	25 MeV

e	f	g	h
134 MeV	1.187 MeV$^{-0.35}$	0.1 MeV	0.282 MeV

2 Comparison of HZETRN formulas with experiment

The HZETRN and BRYNTRN formulas for proton-proton total cross sections are shown as the dashed blue curves in figures 1 and 2 compared to experimental data, shown as the solid triangles, from the Particle Data Group [6]. Figure 1 shows that the apparent overall agreement between theory and experiment is quite good except in the EeV region, where the cosmic ray intensity is very low and therefore of negligible concern for space radiation. The GeV region is of most concern because it is near this region where the cosmic ray spectrum reaches a peak. Therefore, this region is examined in more detail in figure 2. The HZETRN cross section is found to *oscillate* which is not good behavior. In fact, this oscillation is present at most energies. Furthermore, the HZETRN cross section shows some disagreement with experiment in this energy range. Comparison with neutron-proton data is shown in figures 3 and 4. Again, the global fit looks quite good as shown in figure 3. However, when one looks more carefully at the important GeV region as shown in figure 4, one again notes some discrepancies. The aim of the present work is to correct these discrepancies. Their significance is that transport codes may give slightly incorrect results. The input cross sections should always be as accurate as possible.

3 New parametrizations of nucleon-nucleon total cross sections

The GeV region is often referred to as the intermediate energy region. The high energy region is up to 10^9 GeV. Recall that 10^3 GeV = TeV = 10^{12} eV and 10^6 GeV = PeV = 10^{15} eV and 10^9 GeV = EeV = 10^{18} eV and 10^{12} GeV = ZeV = 10^{21} eV. Thus, the region being referred to as the EeV region is of great interest from a pure physics point of view but is not of much interest for space radiation because the cosmic ray flux is very low in

this region. Nevertheless, for the sake of completeness, parametrizations in this region are also studied because data are available. The dominant process in the very high energy region is due to Pomeron exchange, such as flavorless gluons [7]. Parameterizations of this very high energy region has been completed by Collins and Martin [7], Wong [8] and De *et al* [9] and many other authors. This work has been condensed into a formula that provides the best fit between intermediate and high energies by the Particle Data Group [6]. Their cross section parameterization for both pp and pn reactions is

$$\sigma^{PDG}(T_{1l}) \quad = \quad Z^{ab} + B\log^2(s/s_0) + Y_1^{ab}(s_1/s)^{\eta_1} - Y_2^{ab}(s_1/s)^{\eta_2} \tag{10}$$

with the square of the center-of-momentum energy given by

$$s \quad = \quad 2m(m + \sqrt{p_{1l}^2 + m^2}) \tag{11}$$

where m is the nucleon mass and

$$p_{1l} \equiv \sqrt{T_{1l}(T_{1l} + 2m)} \tag{12}$$

In equation (10), the constants $Z^{ab}, B, Y_1^{ab}, Y_2^{ab}$ are in units of mb and s, s_1, s_0 are in units of GeV^2. The values of the constants are listed in Table 2.

Table 2: Constants [6] for Cross Section Formula (10).

Beam	Target	Z (mb)	Y_1 (mb)	Y_2 (mb)	B (mb)	s_1 (GeV^2)	$\sqrt{s_0}$ (GeV)	η_1	η_2
p	p	35.45	42.53	33.34	0.308	1.0	5.38	0.458	0.545
p	n	35.80	40.15	30.00	0.308	1.0	5.38	0.458	0.545

3.1   Proton-Proton cross section

For proton-proton (pp) cross sections the HZETRN formula works well for $p_{lab} < 1.8$ GeV and the Particle Data Group parameterizations work well for $p_{lab} > 4.7$ GeV. Between these energies a better fit is obtained with

$$\sigma_{pp}^{JWN} = \frac{\gamma}{p_{1l}^{0.16}} \tag{13}$$

10

where $\gamma \equiv 52.5$ mb GeV$^{0.16} = 158.547$ mb MeV$^{0.16}$. The final new parameterization is

$$
\begin{align}
\sigma_{pp}^{present} &= \sigma_{pp}^{HZETRN} && \text{for } p_{1l} < 1.8 \text{ GeV} \quad (\text{i.e. } T_{1l} < 1091.74 \text{ MeV}) && (14) \\
&= \sigma_{pp}^{JWN} && \text{for } 1.8 \text{ GeV} \leq p_{1l} \leq 4.7 \text{ GeV} && (15) \\
&= \sigma_{pp}^{PDG} && \text{for } p_{1l} > 4.7 \text{ GeV} \quad (\text{i.e. } T_{1l} > 3854.69 \text{ MeV}) && (16)
\end{align}
$$

Note that these formulas are discontinuous. See figures 1 and 2.

3.2 Neutron-Proton cross section

For proton-proton (pp) cross sections the HZETRN formula works well for $p_{lab} < 1.8$ GeV and the Particle Data Group parameterizations work well for $p_{lab} > 4.7$ GeV. Between these energies a better fit is obtained with

$$
\sigma_{np}^{JWN}(T_{1l}) \;=\; 40 + 10\cos(\alpha\, p_{1l} - 0.943)\exp(-\beta\, p_{1l}^{0.8} + 2) \qquad (17)
$$

with p_{1l} given by (12). The constants α and β are given in Table 3. The final new parameterization of the present work is

$$
\begin{align}
\sigma_{pp}^{present} &= \sigma_{np}^{HZETRN} && \text{for } p_{1l} < 0.5 \text{ GeV} \quad (\text{i.e. } T_{1l} < 124.941 \text{ MeV}) && (18) \\
&= \sigma_{np}^{JWN} && \text{for } 0.5 \text{ GeV} \leq p_{1l} \leq 2.0 \text{ GeV} && (19) \\
&= \sigma_{np}^{PDG} && \text{for } p_{1l} > 2.0 \text{ GeV} \quad (\text{i.e. } T_{1l} > 1271.04 \text{ MeV}) && (20)
\end{align}
$$

Note that these formulas are also discontinuous. See figures 3 and 4.

Table 3: Constants for Cross Section Formula (17).

α	β
0.00369 MeV^{-1}	0.00895741 MeV$^{-0.8}$

4 Conclusions

Nucleon-nucleon total (elastic plus inelastic) cross sections are inputs to space radiation transport codes. The nucleon-nucleon total cross sections currently in use in the transport codes HZETRN and BRYNTRN provide an apparently good global fit to the data. However, when one looks more closely at the important GeV energy range the comparison to data is poor. This energy range is near the maximum intensity of the cosmic ray spectrum and is therefore important for space radiation studies. Improved parameterizations have been developed in the present work, which provide a good fit to data over the entire energy range. The new formulas have been incorporated in the HZETRN code.
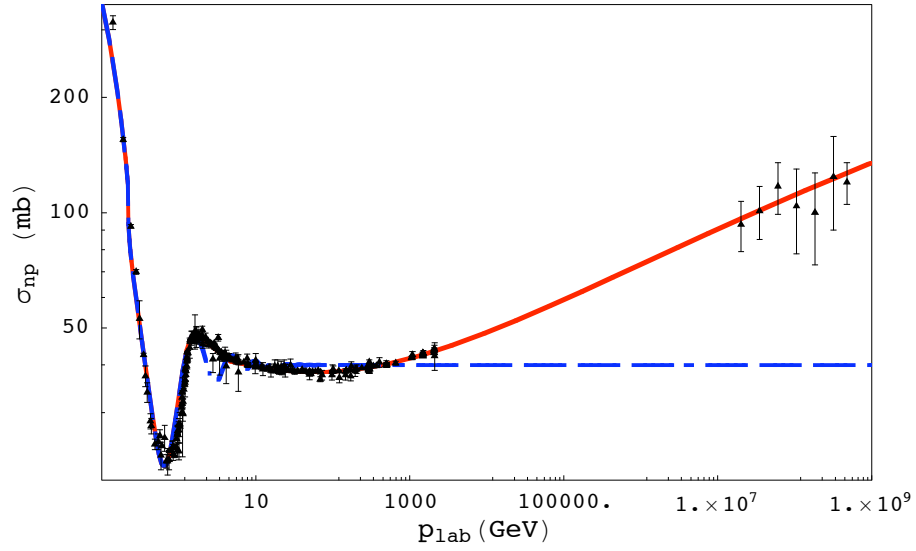
Figure 1: Total proton-proton cross section versus the projectile momentum in the lab frame, p_{lab}. The blue dashed curve is used in HZETRN [4] and the red solid curve is the present work. Experimental data are from Reference [6].
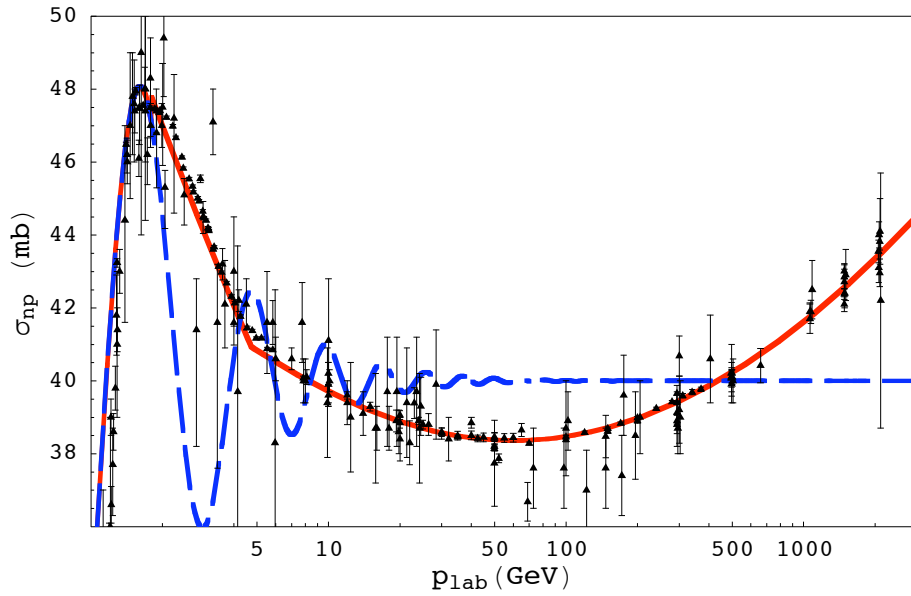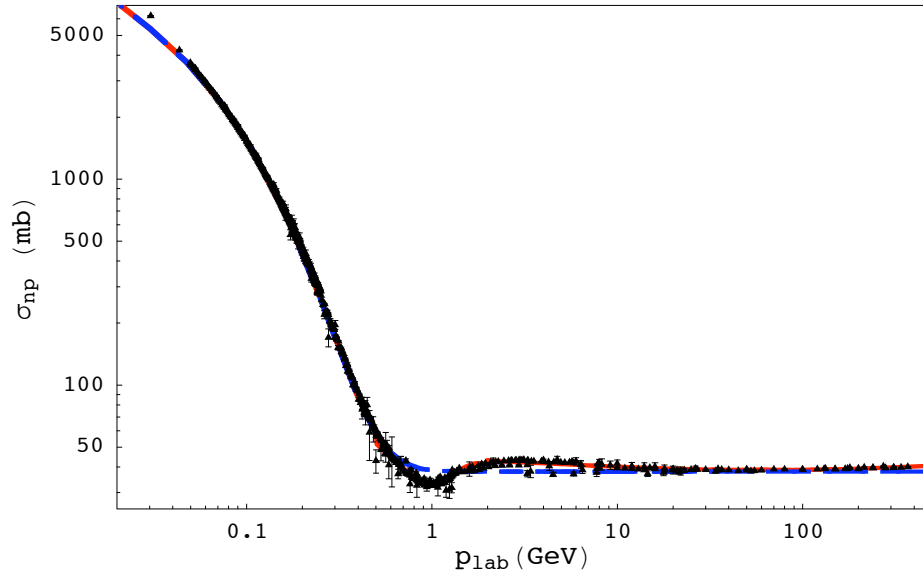

Figure 2: Detail of Fig. 1.

13

Figure 3: Total proton-neutron cross section. The blue dashed curve is used in HZETRN [4] and the red solid curve is the present work. Experimental data are from Reference [6].
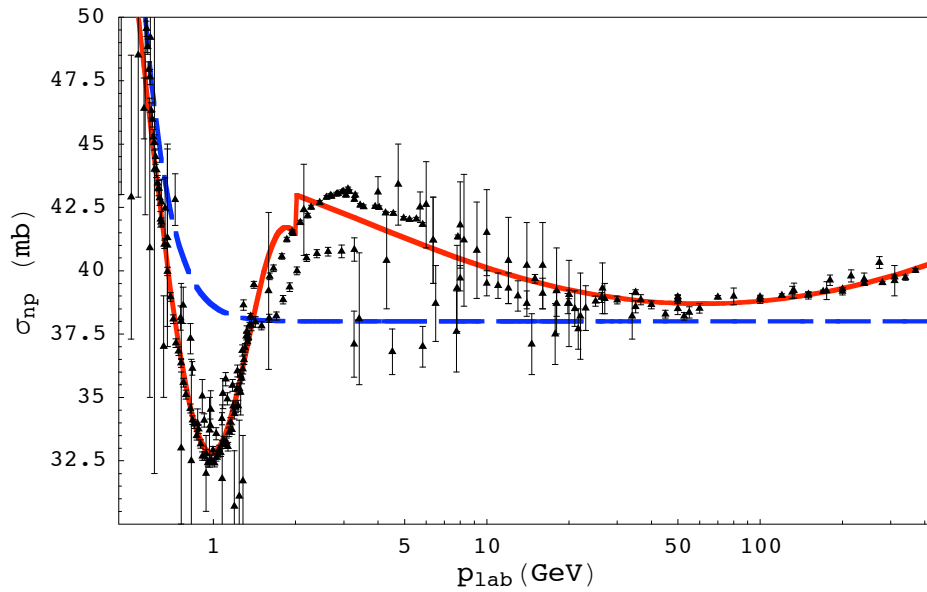

Figure 4: Detail of Fig. 3.

14

References

[1] J. W. Wilson, R. K. Tripathi, G. D. Qualls, F. A. Cucinotta, R. E. Prael, J. W. Norbury, J. H. Heinbockel, and J. Tweed, Advances in Space Research **34**, 1319 (2004).

[2] J. W. Wilson, L. W. Townsend, S. Y. Chun, W. W. Buck, F. Khan, and F. Cucinotta, *BRYNTRN: A baryon transport computer code*, NASA Technical Memorandum 4037 (1988).

[3] J. W. Wilson, L. W. Townsend, J. E. Nealy, S. Y. Chun, B. S. Hong, W. W. Buck, S. L. Lamkin, B. D. Ganapol, F. Khan, and F. A. Cucinotta, *BRYNTRN: A baryon transport model*, NASA Technical Paper 2887 (1989).

[4] J. W. Wilson, L. W. Townsend, W. Schimmerling, G. S. Khandelwal, F. Khan, J. E. Nealy, F. A. Cucinotta, L. C. Simonsen, J. L. Shinn, and J. W. Norbury, *Transport Methods and Interactions for Space Radiations*, NASA Reference Publication No. 1257 (1991).

[5] A. N. Kalinovskii, N. V. Mokhov, and Y. P. Nikitin, *Passage of High-Energy Particles through Matter*, translated by S. J. Amoretty (American Institute of Physics, New York, 1989).

[6] Particle Data Group. J. Phys. G **33**, 1 (2006).

[7] P. D. B. Collins, A. D. Martin, *Hadron Interactions* (Adam Hilger, Bristol, 1984).

[8] C. Y. Wong, *Introduction to High Energy Heavy Ion Collisions* (World Scientific, Singapore, 1994).

[9] B. De, S. Bhattacharyya, and P. Guptaroy, J. Phys. G **27**, 2389 (2001).

1. REPORT DATE *(DD-MM-YYYY)* 01-03-2008	2. REPORT TYPE Technical Publication	3. DATES COVERED *(From - To)*

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Nucleon-Nucleon Total Cross Section	
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)	5d. PROJECT NUMBER
Norbury, John W.	
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER 651549.02.07.01

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
NASA Langley Research Center Hampton, VA 23681-2199	L-19396

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
National Aeronautics and Space Administration Washington, DC 20546-0001	NASA
	11. SPONSOR/MONITOR'S REPORT NUMBER(S) NASA/TP-2008-215116

12. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited
Subject Category 93
Availability: NASA CASI (301) 621-0390

13. SUPPLEMENTARY NOTES
An electronic version can be found at http://ntrs.nasa.gov

14. ABSTRACT

The total proton-proton and neutron-proton cross sections currently used in the transport code HZETRN show significant disagreement with experiment in the GeV and EeV energy ranges. The GeV range is near the region of maximum cosmic ray intensity. It is therefore important to correct these cross sections, so that predictions of space radiation environments will be accurate. Parameterizations of nucleon-nucleon total cross sections are developed which are accurate over the entire energy range of the cosmic ray spectrum.

15. SUBJECT TERMS
Neutron; Space radiation; Cross section

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			STI Help Desk (email: help@sti.nasa.gov)
U	U	U	UU	18	19b. TELEPHONE NUMBER *(Include area code)* (301) 621-0390