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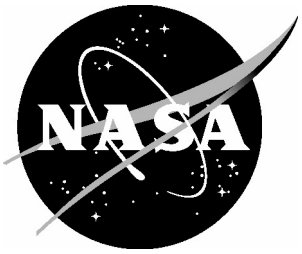

MESTRN: A Deterministic Meson-Muon Transport Code for Space Radiation

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August 2004

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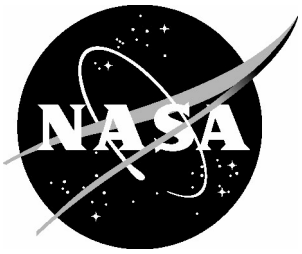

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Frequently Used Nomenclature

Note: the units used in this work are such that $c = 1$ unless otherwise noted. Also, as described in section 3, any function of energy can be written as a function of range instead.

$a_j(E)$	inverse of mean free path of particles of type j and energy E; its dimensions are $\frac{1}{\text{length}}$
$a_{jk}(E, E')$	macroscopic spectral distribution for production of particles of type j and energy E from particles of type k and energy E'; its dimensions are $\frac{1}{\text{length}\cdot\text{energy}}$
$a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')$	macroscopic cross section for production of particles of type j moving in direction $\widehat{\Omega}$ with energy E from particles of type k moving in direction $\widehat{\Omega}'$ with energy E'; its dimensions are $\frac{1}{\text{length}\cdot\text{energy}\cdot\text{solid angle}}$
$\frac{d\sigma}{dE}$	microscopic spectral distributions
E	kinetic energy; unless stated otherwise, all energies in this work are kinetic
$q_j(x, r_j)$	defined by equation (22) or equation (34); it represents a "source" of particles of type j and range r_j
r_j	range of particles of type j, defined by equation (19)
r_p	range of protons

$S_j(E)$	the stopping power of particles of type j and energy E; it has dimensions of $\frac{\text{energy}}{\text{length}}$
$u_j(E)$	same as $a_j(E)$, except it only includes decays
$u_{jk}(E, E')$	same as $a_{jk}(E, E')$ except it only includes production from decays
$u_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')$	same as $a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')$ except it only includes production from decay
$\sigma_j(E)$	same as $a_j(E)$, except it only includes collisions
$\sigma_{jk}(E, E')$	same as $a_{jk}(E, E')$ except it only includes production from collisions
$\sigma_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')$	same as $a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')$ except it only includes production from collisions
$\phi_j(\overrightarrow{x}, E)$	flux of particles of type j located at position $\overrightarrow{x}$, traveling with energy E; its dimensions are $\frac{1}{\text{area}\cdot\text{energy}}$
$\phi_j(\overrightarrow{x}, \widehat{\Omega}, E)$	flux of particles of type j located at position $\overrightarrow{x}$, traveling with velocity $\overrightarrow{v} = v\widehat{\Omega}$ and energy E; its dimensions are $\frac{1}{\text{area}\cdot\text{solid angle}\cdot\text{energy}}$

1 Introduction

To design a spacecraft with optimum shielding, the ability to obtain detailed knowledge of the radiation in the craft's internal environment, when given the external radiation conditions and a set of shielding specifications, is required. The NASA Langley Research Center code HZETRN has been developed for this purpose. This code does not yet include the effects of pions or muons, which are thought to partially account for the discrepancy between HZETRN predictions and a recent experiment [1]. The intent of this paper is to present a method and its implementation as a set of fortran subprograms, called MESTRN, for the inclusion of pions and muons in the NASA transport codes. Results for transport of a cosmic ray spectrum through aluminum and water are also presented.

The development of a transport code for muons and pions was modeled on the existing ion transport methods [2]. A perturbative solution to the Boltzmann transport equation (using the straight-ahead approximation) was used. This solution differs from the solution used for light ions only by making no assumptions about the spectra of produced particles. A subroutine that iterates this solution numerically, so that transport over large distances can be achieved, has been partially integrated into HZETRN. Using this modified code, some results have been obtained for transport through aluminum and water.

In addition to the inputs normally required to run HZETRN, cross sections and stopping powers that describe muon and pion reactions are also required. Muon and pion stopping powers were scaled from proton stopping powers by using a scaling factor deter-

mined by the Bethe theory of stopping power [2]. The production of pions was assumed to be exclusively from nucleus-nucleus and pion-nucleus reactions. A combination of formulas from Ranft [3] and Blattnig et al. [4] was used for parameterizations that describe cross sections for these reactions. Pion decay was assumed to be the only method of muon production. Because this decay is a two-body decay, the rate of muon production can be obtained by conservation of momentum and a Lorentz transformation.

The content of this paper is as follows: Section 2 reviews the stationary Boltzmann transport equation and the continuous slowing down and straight-ahead approximations. Section 3 develops an analytic perturbative solution to the transport equation that is valid for transport over short distances. The latter part of this section shows how the analytic solution can be iterated numerically to achieve transport over arbitrary distances. Section 4 contains the interaction database used as input into the transport equation, including the various pion and muon production cross sections. Section 5 presents results for charged pion and muon fluxes after the initial cosmic ray fluxes were transported through various depths of aluminum and water. Some conclusions are presented in this section as well. Appendix A contains many details of the calculations of the analytic solution to the transport equation that were left out of section 3 for the sake of clarity. Appendix B explains some of the notations that were used for cross sections in this paper. Appendix C contains the details of derivations of macroscopic muon cross sections that were listed in section 4. Appendix D describes the MESTRN Fortran subprograms, and appendix E contains these subprograms in their entirety.

2 Derivation of the Transport Equation

Consider a region of space filled with matter described by the appropriate cross sections. Further, consider a spherical volume element of radius δ centered about position $\overrightarrow{x}$. Define $\phi_j(\overrightarrow{x}, \widehat{\Omega}, E)$ to be the flux of particles of type j located at position $\overrightarrow{x}$, traveling with velocity $\overrightarrow{v} = v\widehat{\Omega}$ and energy E. The relation between flux and the distribution function f is discussed in Reif [5], flux is related to f by a factor of velocity. The particle fluxes are assumed to be time independent, including only stationary states. Neglecting time dependence, the units of the flux $\phi_j(\overrightarrow{x}, \widehat{\Omega}, E)$ are $\frac{1}{\text{area}\cdot\text{solid angle}\cdot\text{energy}\cdot\text{time}}$.

Now, consider the flow of particles through this volume element. The number of type j particles leaving a surface element $\delta^2 d\widehat{\Omega}$ of the sphere in a direction $\widehat{\Omega}$, with energy E, is $\delta^2 d\widehat{\Omega}\phi_j(\overrightarrow{x} + \delta\widehat{\Omega}, \widehat{\Omega}, E)$. Similarly, $\delta^2 d\widehat{\Omega}\phi_j(\overrightarrow{x} - \delta\widehat{\Omega}, \widehat{\Omega}, E)$ is the number entering the opposite part of the sphere. Figure 1 gives a sketch of the situation considered here. The number of particles entering the sphere differs from the number leaving by the gains and losses due to interactions with the material and the decay of unstable particles. Any interaction of the radiation with itself is assumed to be negligible. These considerations can be represented in a mathematical formula as [2, 6]

$$\begin{aligned} \delta^2 d\widehat{\Omega}[\phi_j(\overrightarrow{x} + \delta\widehat{\Omega}, \widehat{\Omega}, E) - \phi_j(\overrightarrow{x} - \delta\widehat{\Omega}, \widehat{\Omega}, E)] = \\ \delta^2 d\widehat{\Omega} \int_{-\delta}^{\delta} dl \int dE' d\widehat{\Omega}' \sum_k a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')\phi_k(\overrightarrow{x} + l\widehat{\Omega}, \widehat{\Omega}', E') \\ -\delta^2 d\widehat{\Omega} \int_{-\delta}^{\delta} dl \phi_j(\overrightarrow{x} + l\widehat{\Omega}, \widehat{\Omega}, E)a_j(E) \end{aligned} \tag{1}$$

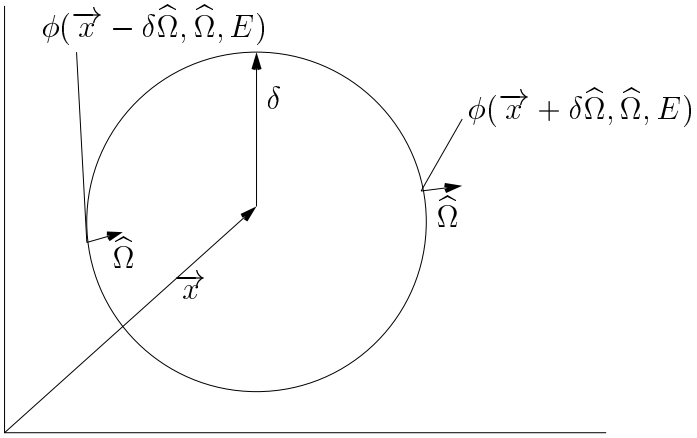

Figure 1: Spherical volume element of radius δ with fluxes of particles at opposing positions on labeled surface. These two fluxes are equal if no interaction occurs while particles move length δ through sphere.

The limits of the integrals over dE' and $d\widehat{\Omega}'$ are constrained by conservation of momentum and energy, or equivalently, dE' and $d\widehat{\Omega}'$ range over all values, and the cross sections are constrained by conservation of momentum and energy.

Radiation is treated as a flux of classical particles; interference effects between the radiation and the bulk medium are therefore neglected. This type of interference is not expected to have much of an effect on the processes considered here because of the large energies of the particles in question. Quantum effects are, however, the main consideration when calculating cross sections for interactions between the radiation and individual nuclei.

Equation (1) can be understood as a statement of conservation of flux. The change

in the amount of flux (the left-hand side) is equal to the amount destroyed (second term on right-hand side) subtracted from the amount created (first term on right-hand side); $\phi_k(\overrightarrow{x} + \delta\widehat{\Omega}, \widehat{\Omega}, E)a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')$ is the number of particles per volume, energy, and solid angle of energy E (and moving in the direction $\widehat{\Omega}$) that are created by particles of type k moving in the direction $\widehat{\Omega}'$ with the energy E'. This statement defines what is meant by the *macroscopic* production cross section $a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')$, which has the units of $\frac{1}{\text{length}\cdot\text{energy}\cdot\text{solid angle}}$. An example of this type of process is a proton (particle k) colliding with the medium and producing a pion (particle j). Particles can be created either from the decay of unstable particles or from interaction with the medium. For example, pions can be created from the collision of heavy ions with a medium or by the decay of kaons. Multiple reactions can produce these types of results, making the appropriate cross sections inclusive.

$$a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E') = \sigma_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E') + u_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E') \tag{2}$$

where σ_{jk} represents production from collisions, and u_{jk} represents production from decay.

By integrating over E' and $\widehat{\Omega}'$ and summing over k, incoming particles of all energies, angles, and types are considered, leaving the number of particles of type j produced per volume. To get the number of particles produced, we integrate over the path through the sphere $\int_{-\delta}^{\delta} dl$ and multiply by the surface element $\delta^2 d\widehat{\Omega}$ (see fig. 2); $a_j(E)\phi_k(\overrightarrow{x} + \delta\widehat{\Omega}, \widehat{\Omega}, E)$ is the number of particles of type j moving in the direction $\widehat{\Omega}$ with energy E per volume that are lost. What is meant by lost is that they are no longer of type j, or they are

no longer moving in the same direction with the same energy. Furthermore, $a_j(E)$ is the inverse of the mean free path and has the dimensions length^{-1}. It is a value of unity divided by the distance a particle of type j and energy E will travel before having an interaction or decaying and becoming "lost"; $a_j(E)$ can be split into two components, one for decays and one for collisions.

$$a_j(E) = u_j(E) + \sigma_j(E) \tag{3}$$

The term $u_j(E)\phi_j(E)$ is the number of decays per unit volume per unit energy, and $\sigma_j(E)\phi_j(E)$ is the number of collisions per unit volume per unit energy. The symbol $\sigma_j(E)$ is the total *macroscopic* interaction cross section (inelastic plus elastic) for particles of type j that interact with the medium. For the sake of simplicity, these cross sections are considered independent of position; the medium is assumed to be constant. See section 4 for more details on cross sections and decay lengths, particularly the relationship between microscopic and macroscopic cross sections.

The symbol δ is the radius of the region under consideration, and it can be made arbitrarily small, allowing the fluxes to be expanded in a three-dimensional Taylor series around the vector $\overrightarrow{x}$.

$$\phi(\overrightarrow{x} \pm l\widehat{\Omega}, \widehat{\Omega}, E) = \phi(\overrightarrow{x}, \widehat{\Omega}, E) \pm l\widehat{\Omega} \cdot \overrightarrow{\bigtriangledown}\phi(\overrightarrow{x}, \widehat{\Omega}, E) + O(\delta^2) \tag{4}$$

Therefore,

$$\int_{-\delta}^{\delta} dl\, \phi(\overrightarrow{x} + l\widehat{\Omega}, \widehat{\Omega}, E) = 2\delta\phi(\overrightarrow{x}, \widehat{\Omega}, E) + O(\delta^3) \tag{5}$$

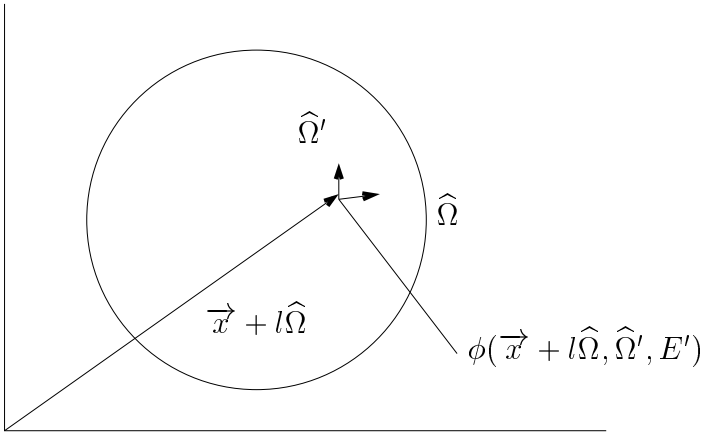

Figure 2: At point $\overrightarrow{x} + l\widehat{\Omega}$, particles moving in direction $\widehat{\Omega}'$ create particles moving in direction $\widehat{\Omega}$.

Equation (4) is a three-dimensional Taylor series expansion, in which only terms up to order δ are kept. The operator $\widehat{\Omega} \cdot \overrightarrow{\bigtriangledown}$ is the spatial derivative in the direction $\widehat{\Omega}$. For example, if $\widehat{\Omega}$ is pointed in the $\widehat{r}$ direction, then $\widehat{\Omega} \cdot \overrightarrow{\bigtriangledown} = \frac{\partial}{\partial r}$. Equation (5) is the integral of the fluxes given by equation (4). The terms of odd order in l integrate to 0, so there is no term of order δ^2 in equation (5).

The next step in the derivation is the substitution of the above two equations into equation (1) and the simplification of the result. A three-dimensional transport equation results [2, 6],

$$\begin{aligned} \phi_j(\overrightarrow{x}, \widehat{\Omega}, E) \;+\; & \delta\widehat{\Omega} \cdot \overrightarrow{\bigtriangledown}\phi_j(\overrightarrow{x}, \widehat{\Omega}, E) - [\phi_j(\overrightarrow{x}, \widehat{\Omega}, E) - \delta\widehat{\Omega} \cdot \overrightarrow{\bigtriangledown}\phi_j(\overrightarrow{x}, \widehat{\Omega}, E)] + O(\delta^2) \\ = \; & \int dE'\, d\widehat{\Omega}' \sum_k a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E')[2\delta\phi_k(\overrightarrow{x}, \widehat{\Omega}', E')] \\ & - [2\delta\phi_j(\overrightarrow{x}, \widehat{\Omega}, E)]a_j(E) + O(\delta^3) \end{aligned}$$

which reduces to [2, 6]

$$\begin{aligned} \widehat{\Omega} \cdot \overrightarrow{\nabla} \phi_j(\overrightarrow{x}, \widehat{\Omega}, E) &= \int dE' d\widehat{\Omega}' \sum_k a_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E') \phi_k(\overrightarrow{x}, \widehat{\Omega}', E') \\ &\quad - \phi_j(\overrightarrow{x}, \widehat{\Omega}, E) a_j(E) + O(\delta) \end{aligned} \tag{6}$$

The limit as $\delta \to 0$ can now be taken, making the terms of order δ vanish.

2.1 Continuous Slowing Down Approximation

Cross sections σ_{jk} and σ_j describe all the relevant processes except those that involve decay. A simplification can be made if the atomic and nuclear components of these cross sections are treated separately. Examples of atomic cross sections are atomic or molecular transitions of the medium caused by the radiation. The nuclear cross sections describe collisions between the nuclei of the incoming radiation and the nuclei in the material under consideration. More precisely, what is meant by "atomic" is any process that can be approximately described by equation (9), and "nuclear" refers to all processes not accounted for elsewhere. The separation is made as follows:

$$\sigma_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E') = \sigma^n_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E') + \sigma^{at}_{jk}(\widehat{\Omega}, \widehat{\Omega}', E, E') \tag{7}$$

and

$$\sigma_j(E) = \sigma^n_j(E) + \sigma^{at}_j(E) \tag{8}$$

where superscripts n and at refer to the nuclear and atomic components, respectively.

No significant deflection occurs in a collision with an atomic electron when the following conditions are met: the particle is much more massive than an electron ($m_j \gg m_{\text{electron}}$), and the energy transferred is much smaller than the original energy ($E \gg \epsilon_n$) and is below particle production threshold. When these conditions are satisfied, the following approximate form for σ_{jk}^{at} can be assumed [2, 6].

$$\sigma_{jk}^{at}(\widehat{\Omega}, \widehat{\Omega}', E, E') \approx \sum_n \sigma_{jn}^{at}(E)\delta^2(\widehat{\Omega} \cdot \widehat{\Omega}' - 1)\delta_{jk}\,\delta(E + \epsilon_n - E') \tag{9}$$

where n labels the excitation level, ϵ_n is the energy of the excited state n, and $\sigma_{jn}^{at}(E)$ is the cross section for excitation to the nth state. The term $\delta(E+\epsilon_n-E')$ enforces conservation of energy, and the term δ_{jk} preserves particle type. The angular delta function conserves direction, and is defined by the equation $\int f(\widehat{\Omega}')\delta^2(\widehat{\Omega} \cdot \widehat{\Omega}' - 1)d\widehat{\Omega}' = f(\widehat{\Omega})$, where $\widehat{\Omega} \cdot \widehat{\Omega}'$ is the scalar product of two unit vectors.

For the sake of clarity, only the terms involving atomic cross sections from equation (6) will now be considered, ignoring terms including nuclear components of the cross sections, decay lengths, and spatial derivatives. The terms of interest from equation (6) are

$$\sum_k \int d\widehat{\Omega}' dE' \sigma_{jk}^{at}(\widehat{\Omega}, \widehat{\Omega}', E, E')\phi_k(\overrightarrow{x}, \widehat{\Omega}', E') - \sigma_{jn}^{at}(E)\phi_j(\overrightarrow{x}, \widehat{\Omega}, E)$$

Equation (9) then implies that [2, 6]

$$\begin{aligned}\sum_k \int d\widehat{\Omega}'\, dE' \sigma_{jk}^{at}(\widehat{\Omega}, \widehat{\Omega}', E, E')\phi_k(\overrightarrow{x}, \widehat{\Omega}', E') - \sigma_{jn}^{at}(E)\phi_j(\overrightarrow{x}, \widehat{\Omega}, E) \\ = \sum_n \sigma_{jn}^{at}(E + \epsilon_n)\phi_j(\overrightarrow{x}, \widehat{\Omega}, E + \epsilon_n) - \sigma_j^{at}(E)\phi_j(\overrightarrow{x}, \widehat{\Omega}, E)\end{aligned} \tag{10}$$

A series expansion of $\sigma_{jn}^{at}(E+\epsilon_n)$, keeping terms up to order ϵ_n, can now be made [2, 6].

$$\begin{aligned}\sum_n \sigma_{jn}^{at}(E+\epsilon_n)\phi_j(\overrightarrow{x},\widehat{\Omega},E+\epsilon_n) &- \sigma_j^{at}(E)\phi_j(\overrightarrow{x},\widehat{\Omega},E) = \sum_n \sigma_{jn}^{at}(E)\phi_j(\overrightarrow{x},\widehat{\Omega},E) \\ +\sum_n \epsilon_n \frac{\partial}{\partial E}[\sigma_{jn}^{at}(E)\phi_j(\overrightarrow{x},\widehat{\Omega},E)] &+ O(\epsilon_n^2) - \sigma_j^{at}(E)\phi_j(\overrightarrow{x},\widehat{\Omega},E) \\ &\approx \frac{\partial}{\partial E}[S_j(E)\phi_j(\overrightarrow{x},\widehat{\Omega},E)] \qquad (11)\end{aligned}$$

where the stopping power is defined as $S_j(E) \equiv \sum_n \epsilon_n \sigma_{jn}^{at}(E)$. The cross section for an arbitrary excitation $\sigma_j^{at}(E)$ is simply the sum of the cross sections for a particular excitation $\sigma_{jn}^{at}(E)$. Thus [2, 6],

$$\sigma_j^{at}(E) = \sum_n \sigma_{jn}^{at}(E) \qquad (12)$$

because the first and third terms on the right-hand side of equation (11) have canceled. The term $\sigma_{jn}^{at}(E)$ is a macroscopic cross section and has units of length^{-1}, not of area. Stopping power therefore has units of $\frac{\text{energy}}{\text{length}}$ and can be interpreted as an average energy loss per unit distance traveled. Keeping terms only up to order ϵ_n is justified because $E \gg \epsilon_n$, but in some situations, such as nearly mono-energetic beams, higher order terms can become important [2].

The continuous slowing down approximation is equivalent to the situation where particles continuously lose energy as they travel through a medium, hence the name. The rate of energy loss is averaged over all different possible excitation levels n, and is given by the stopping power $S_j(E)$. See [2, 7, 8, 9, 10, 11] for more details on these approximations, and on stopping power in general.

Substituting equation (11) into equation (6) and rearranging the terms results in a three-dimensional time independent transport equation in the continuous slowing down approximation [2, 6],

$$\left[\widehat{\Omega}\cdot\overrightarrow{\bigtriangledown}-\frac{\partial}{\partial E}S_j(E)+a_j(E)\right]\phi_j(\overrightarrow{x},\widehat{\Omega},E)$$
$$=\int dE'\,d\widehat{\Omega}'\sum_k a_{jk}(\widehat{\Omega},\widehat{\Omega}',E,E')\phi_k(\overrightarrow{x},\widehat{\Omega}',E') \tag{13}$$

where all of the cross sections are now purely nuclear. The superscript n was dropped to simplify the notation. Also recall that the derivative $\frac{\partial}{\partial E}$ acts on both $S_j(E)$ and $\phi_j(E)$, as written on the right-hand side of equation (11).

2.2 Straight-Ahead Approximation

When particles are produced mainly in the direction of the primary flux, a further simplification can be made. This simplification, commonly referred to as the straight ahead approximation, reduces equation (13) from three spatial dimensions to one spatial dimension.

The straight-ahead approximation is valid when the particle production cross sections are highly peaked in the forward direction, which is when there is little change in direction due to collisions. This criterion is expressed in the following equation:

$$a_{jk}(\widehat{\Omega},\widehat{\Omega}',E,E')\approx a_{jk}(E,E')\delta^2(\widehat{\Omega}\cdot\widehat{\Omega}'-1) \tag{14}$$

This equation is often accurate in high energy collisions, in which conservation of momentum keeps particles moving mainly in the forward direction. Because the direction of

radiation flow is not being greatly changed by collisions, only a single spatial dimension is needed. Thus,

$$\phi(\overrightarrow{x}, \widehat{\Omega}, E) \rightarrow \phi(x, E) \tag{15}$$

and

$$\widehat{\Omega} \cdot \overrightarrow{\bigtriangledown} \rightarrow \frac{\partial}{\partial x} \tag{16}$$

resulting in [2, 6, 12, 13, 14]

$$\Big[\frac{\partial}{\partial x} - \frac{\partial}{\partial E} S_j(E) + a_j(E)\Big] \phi_j(x, E) = \int dE' \sum_k a_{jk}(E, E') \phi_k(x, E') \tag{17}$$

Recall that $a_j(E) \equiv \sigma_j(E) + u_j(E)$. Thus, in the above equation $\frac{\partial}{\partial x}$ and σ have the same dimensions. In the code HZETRN, the "length" x has units of $\frac{\mathrm{g}}{\mathrm{cm}^2}$, and the macroscopic cross section σ has the corresponding units $\frac{\mathrm{cm}^2}{\mathrm{g}}$. The relation between the macroscopic and microscopic cross sections is

$$\sigma = \overline{\rho} \sigma_{\mathrm{microscopic}} \tag{18}$$

for each target species, where $\rho \equiv \frac{\mathrm{Number\,of\,particles}}{\mathrm{Mass}}$.

The straight-ahead approximation simplifies the transport equation by reducing it to a single spatial dimension. A three-dimensional problem can now be solved by superposition. Because of linearity, a superposition of the solutions to the single dimensional equation (17) is the solution to the full three-dimension equation (13). However, some assumptions must be made on the boundary conditions, which can also introduce some error as is described in Wilson et al. [2].

3 Solution Method

Equation (17) is an integro-differential equation. The first step in our solution method is to transform equation (17) into a pure integral equation, integrating to get rid of the derivatives. Second, a series of variable transformations are made, simplifying the resulting equation. A perturbative solution accurate only over short distances is then obtained, but it can be iterated numerically to obtain results for longer distances.

3.1 Analytic Solution

First, define the residual range [2, 6, 14, 15]

$$r_j(E) \equiv \int_0^E \frac{dE'}{S_j(E')} \tag{19}$$

which implies

$$\frac{dr_j}{dE} = \frac{1}{S_j(E)} \tag{20}$$

Physically, r_j represents the average distance a particle of type j would travel if only atomic interactions (stopping power) were considered.

Now, define a new function [2, 6, 14, 15],

$$\psi_j(x, r_j) \equiv S_j(E)\phi_j(x, E) \tag{21}$$

Physically ψ_j, which has units of $\frac{\text{energy}}{\text{volume}}$, represents the amount of energy lost at location x by particles of type j and range r_j [2].

By using equations (19)-(21) in equation (17), one can show [2, 6, 14]

$$\Big[\frac{\partial}{\partial x} - \frac{\partial}{\partial r_j} + a_j(r_j)\Big]\psi_j(x, r_j) = q_j(x, r_j) \tag{22}$$

where

$$q_j(x, r_j) = S_j(r_j) \sum_k \int dr_k a_{jk}(r_j, r_k)\psi_k(x, r_k)$$

represents the creation of particles of type j, at the position x, with range r_j.

Inverting equation (22) results in [2, 14]

$$\begin{aligned} \psi_j(x, r_j) &= \exp\Big[-\frac{1}{2}\int_{-x-r_j}^{x-r_j} d\eta'' a_j\big(\frac{x + r_j - \eta''}{2}\big)\Big]\psi_j(0, x + r_j) \\ &\quad + \frac{1}{2}\int_{-x-r_j}^{x-r_j} d\eta' \exp\Big[-\frac{1}{2}\int_{\eta'}^{x-r_j} d\eta'' a_j\big(\frac{x + r_j - \eta''}{2}\big)\Big] q_j(\eta') \end{aligned} \tag{23}$$

where $\eta \equiv x - r$. Equation (23) is an exact solution to equation (22). No approximation has yet been made. If h is a small distance, then ψ_j at position $x + h$ can be determined as a function of ψ_j at position x. The following equation is the result:

$$\begin{aligned} \psi_j(x + h, r_j) &\approx \exp(-ah)\Big\{\psi_j(x, r_j + h) + q_j(x, r_j + h)\frac{\exp(ah) - 1}{a} \\ &\quad + \Delta q_j\Big[\exp(ah)\big(\frac{h}{a} - \frac{1}{a^2}\big) + \frac{1}{a^2}\Big]\Big\} \end{aligned} \tag{24}$$

where $\Delta q_j \approx \frac{q_j(x, r_j+h) - q_j(x-h, r_j)}{h}$ and $a \equiv a_j(r_j)$. Derivations of equations (23) and (24) can be seen in appendix A.

Equation (24) is an approximate algebraic solution to equation (17) which is valid for small h. However, the solution is for the function ψ not ϕ (recall eq. (21)). The flux ϕ can be obtained by simply dividing by the appropriate stopping power. For particle transport

that involves distances greater than a small value h, equation (24) can be iterated in steps of h in a manner that will be discussed subsequently.

3.2 Numerical Solution to Transport Equation

The first step in the numerical solution is to make all quantities a function of energy per amu (atomic mass unit) rather than energy. Equation (17) then becomes [2, 6, 14],

$$\Big[\frac{\partial}{\partial x} - \frac{1}{A_j}\frac{\partial}{\partial E}S_j(E) + a_j(E)\Big]\phi_j(x,E) = \int dE' \sum_k a_{jk}(E,E')\phi_k(x,E') \tag{25}$$

where A_j is the mass of the jth particle in amu, which is approximately equal to the atomic number in the case of nucleons. Now define

$$\tilde{S}_j(E) = \frac{1}{A_j}S_j(E) \tag{26}$$

$$\tilde{r}_j(E) = \int_0^E \frac{dE'}{\tilde{S}_j(E')} \tag{27}$$

Equation (25) becomes exactly the same as equation (17), except with $\tilde{S}$ rather than S. The rest of the analysis follows through exactly the same as in section 3.1, except with $\tilde{S}$ rather than S in equation (24).

Using the Bethe theory for stopping power implies [2, 7, 13]

$$\tilde{S}_j(E) \approx \frac{\nu_j}{\nu_k}\tilde{S}_k(E) \tag{28}$$

$$\nu_j\tilde{r}_j(E) \approx \nu_k\tilde{r}_k(E) \tag{29}$$

where $\nu_i = \frac{Z_i^2}{A_i}$ and Z_i is charge. However, the Bethe theory, and consequently the previous equations, are only accurate for limited energy ranges [2, 7, 8, 13]. The simple relation

between ranges allows equation (24) to be written as a function of the range of one type of particle. By choosing this particle to be the proton ($\nu_p = 1$), equation (24) becomes

$$\begin{aligned}\psi_j(x+h, r_p) &\approx \exp(-ah)\Big\{\psi_j(x, r_p+\nu_j h) + q_j(x, r_p+\nu_j h)\frac{\exp(ah)-1}{a} \\ &\quad +\Delta q_j\Big[\exp(ah)\big(\frac{h}{a}-\frac{1}{a^2}\big)+\frac{1}{a^2}\Big]\Big\} \qquad (30)\end{aligned}$$

where

$$a = a_j(r_p) \qquad (31)$$

$$\Delta q_j \approx \frac{q_j(x, r_p+\nu_j h) - q_j(x-h, r_p)}{h} \qquad (32)$$

and where all stopping powers (S) are now ($\tilde{S}$). In other words, the function ψ_j is now

$$\psi_j(x, r) \equiv \tilde{S}_j(E)\phi_j(x, E) \qquad (33)$$

and the new source term is

$$\begin{aligned}q_j(x, r_p) &= \tilde{S}_j(r_p)\sum_k \int dr_k a_{jk}(r_p, r_k)\psi_k(x, r_k) \\ &= \tilde{S}_j(r_p)\sum_k \int dr'_p a_{jk}(r_p, r'_p)\tilde{S}_p(r_p)\phi_k(x, r'_p) \qquad (34)\end{aligned}$$

When both $a \ll 1$ and $ah \ll 1$, the quantity $\exp(ah)(\frac{h}{a}-\frac{1}{a^2})+\frac{1}{a^2}$ in equation (30) can be sensitive to numerical round-off error. By expanding the exponential term in an infinite series, it can be shown that

$$\exp(ah)\Big(\frac{h}{a}-\frac{1}{a^2}\Big)+\frac{1}{a^2} = \frac{h^2}{2}\Big[1+\frac{2}{3}ah+\frac{1}{12}(ah)^2\exp(ah)\Big] \qquad (35)$$

This relation is exact, and the term on the right-hand side of the equal sign is insensitive to round-off error when $a \ll 1$ and $ah \ll 1$.

To simplify the numerical implementation, make the further approximation in eq. (30)

$$q_j(x, r + \nu h) \approx q_j(x, r) \tag{36}$$

This term represents particles produced in collisions at position x with a range $r + \nu h$. Evaluating this term at range r neglects continuous slowing down for particles produced until the step after it is produced. For example, a pion is produced somewhere between x and $x + h$. Slowing down is not accounted for until $x + h$. Also, make another approximation for Δq which is a first order approximation to the derivative of the term q. This approximation is similar to the previous one. Evaluate all terms at the same r rather than r and $r + \nu h$. These approximations have little effect on the results because h is small, and they greatly simplify the numerical implementation.

Because everything is a function of one range, equation (30) can be put on a single grid of position and range, then iterated numerically in steps of h. Enter the initial ψ_j into the right hand of equation (30), and ψ_j (at distances h away) will be given by the left-hand side. Numerical interpolation can be used to calculate values of ψ_j that do not lie directly on grid points. This process can be repeated to achieve transport over arbitrary distances. To calculate the fluxes for a three-dimensional geometry, simply calculate the single dimensional solution (eq. (30)) along rays leading to the points where knowledge of the radiation flux is desired, summing the results.

Equation (30) gives the particle fluxes at position $x + h$ as a function of the particle fluxes at position x, allowing this equation to be iterated numerically to achieve transport

over arbitrary distances. To numerically iterate, do the following:

(1) Set up a discreet distance-range grid for the fluxes to be evaluated at each grid point. Since the solution depends only on position and proton range, the same grid can be used for all particle types.

(2) Enter initial fluxes for all range values at $x = 0$.

(3) Solve for fluxes at $x = h$ by using equation (30). This step will need to use a numerical interpolation routine because the particle fluxes will be needed at values of range not lying directly on grid points.

(4) Repeat step 3 until the desired depth is reached.

3.3 Calculating the Source Term Numerically

Because the source term (eq. (34)) appears as a function of range in the transport equation, and particle production cross sections are never calculated as functions of range (note that range depends on the material), there can be some difficulty in numerically evaluating equation (34). In HZETRN, there is both a proton range array and a proton energy array. The range in the ith slot of the range array corresponds to the energy of the ith slot in the energy array so that

$$\begin{aligned} q_j(x, r_i) &= \tilde{S}_j(r_i) \sum_k \int_{E_{\min}}^{E_{\max}} dr' \tilde{S}_p(r') a_{jk}(r_i, r') \phi_k(x, r') \\ &= \tilde{S}_j(r_i) \sum_k \int_{E_{\min}}^{E_{\max}} dE' a_{jk}(E_i, E') \phi_k(x, E') \end{aligned} \tag{37}$$

where the integral is presently over the entire energy grid. Because $\psi_k(x, E')$ is only stored in memory at discrete energy points, a numerical interpolation routine is needed to calculate the fluxes in the evaluation of the integral. The integral is calculated by using a Gaussian integration routine.

3.4 Limits of Integration

The values for $E_{\min}$ and $E_{\max}$ are different for pions than they are for muons. Pions are assumed to be created entirely from collisions of nuclei on nuclei and pions on nuclei. The pion production threshold for proton-proton collisions is approximately 290 MeV. Nucleus-Nucleus collisions are scaled from proton-proton collisions, so the threshold for nucleus-nucleus collisions is just 290 $\frac{\text{MeV}}{\text{amu}}$. Pion production threshold in pion-nucleus reactions is approximately 170 MeV or 1545 $\frac{\text{MeV}}{\text{amu}}$ [16] when neglecting elastic scattering and charge exchange reactions. The threshold is defined as the minimum energy required to produce a particle. $E_{\min}$ is the minimum energy needed to produce a particle with a particular energy. $E_{\min}$ can be approximated as the kinetic energy of the projectile or 290 $\frac{\text{MeV}}{\text{amu}}$, whichever is greater. For pion production, there is, in principle, no limit on $E_{\max}$; however, a transport code will only transport particles of a finite energy range. For the case of space radiation, the initial particle fluxes fall off sharply at high energies, so some cut off energy is chosen and particles above that energy are not transported. Because the fluxes above this energy will be quite small, the effect of this cut off should be negligible

for the purposes of radiation shielding. $E_{\max}$ for the pion is simply identical to this cut off energy.

Muons are considered to be created entirely from pion decay, so $E_{\min}$ and $E_{\max}$ are the minimum and maximum energies a pion can have and still decay into a muon of energy E_i. Since pion decay is a two-body decay, the muon energy is uniquely determined by the pion energy as described in appendix C. To calculate these limiting values, consider the Lorentz transformation muon energy from the pion rest frame to the lab frame

$$E_\mu = \gamma_\pi(E^* + m_\mu - \beta p^* \cos\theta) - m_\mu \tag{38}$$

where E_μ is muon energy in the lab frame, and θ is the angle between the direction of the pion and the muon, $\gamma = \frac{E+m}{m}$ is the Lorentz factor, and $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$ is a speed. E^* is the muon energy in the pion rest frame, and p^* is the corresponding momentum. Remember that all energies are kinetic unless otherwise noted. Pion energy will be minimum when the pion moves in the same direction as the muon, when $\theta = 0$. Pion energy will be maximum when the pion moves in the opposite direction as the muon, when $\theta = \pi$. So,

$$\begin{aligned} E_\mu &= \gamma_\pi(E^* + m_\mu \pm \beta_\pi p^*) - m_\mu \\ &= \gamma_\pi(E^* + m_\mu) \pm p^* \sqrt{\gamma_\pi^2 - 1} - m_\mu \end{aligned} \tag{39}$$

which implies

$$\left[E_\mu + m_\mu - \gamma_\pi(E^* + m_\mu)\right]^2 = (\gamma_\pi^2 - 1)(p^*)^2 \tag{40}$$

which has two solutions

$$\gamma_\pi = \frac{(E^* + m_\mu)(E_u + m_\mu) \pm p^* p_\mu}{m_\mu^2} \tag{41}$$

The solutions corresponding to the maximum and minimum pion momenta are

$$E_{\max/\min} = \frac{(E^* + m_\mu)(E_u + m_\mu) \pm p^* p_\mu}{m_\mu^2} m_\pi \tag{42}$$

4 Database

For the transport of pions and muons, all relevant interactions involving these particles need to be considered. The quantities describing these interactions when using the straight ahead and continuous slowing down approximations are as follows:

(1) The inverse mean free paths $a(r)$

(2) Stopping powers $S(r)$

(3) Macroscopic particle production spectral distributions $a_{jk}(r, r)$ for all processes that produce significant numbers of pions and muons

(4) The initial pion and muon spectrum, which for cosmic rays is 0

These quantities will now be discussed in detail.

4.1 Inverse Mean Free Paths

The inverse mean free path of a particle can be expressed as the sum of the total macroscopic cross section (elastic plus inelastic) and the inverse decay length.

$$a_j(r) = \sigma_j(r) + u_j(r) \tag{43}$$

Particle loss due to decay is described by the function $u_j(r)$, where $u_j(r)\phi_j(r)$ is the number of decays, per unit volume, of particles of type j and range r. To calculate this function, simply look up the lifetime of the particle in question [11], and perform a Lorentz transformation from the rest frame of the particle to the rest frame of the medium. Thus the inverse decay length is [12]

$$u_j(r) = \frac{1}{\beta_j(r) c\tau_j \gamma(r)} \tag{44}$$

where $\gamma = \frac{E_j(r)+m_j c^2}{m_j c^2}$ is the Lorentz factor, when E_j is the kinetic energy of particle j, and $\beta \equiv \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2(r)}}$ gives the velocity of the particle, and τ_j is its lifetime. The term c is written explicitly here and is not set equal to 1 because $c\tau$ is the quantity usually tabulated [11]. Elsewhere, $c = 1$ is assumed.

The *macroscopic* cross section $\sigma_j(r)$ represents all nuclear processes by which particles of type j either change species, energy, or direction; $\sigma_j(r)$ therefore consists of both the elastic and the inelastic cross sections for all nuclear interactions with the medium. The cross section is macroscopic, and it will consist of the sum of all microscopic interactions weighted by the relevant densities. For a medium of N different types of nuclei, each with

a number density ρ_n (units $\frac{1}{\text{volume}}$),

$$\sigma_j(r) = \sum_{n=1}^{N} \rho_n [\sigma_{jn}^{\text{elastic}}(r) + \sigma_{jn}^{\text{inelastic}}(r)] \tag{45}$$

where $\sigma_{jn}^{\text{elastic}}(r)$ and $\sigma_{jn}^{\text{inelastic}}(r)$ are the *microscopic* cross sections for collisions of particles of type j with nuclei of type n. Note that the discreet nature of the medium on the microscopic level is, in effect, averaged over by the introduction of the continuous macroscopic density ρ.

Because muons only interact by the electroweak force, their short-range interactions with nuclei can be neglected, and their long-range electromagnetic interaction can be included in stopping power; therefore,

$$\sigma_{\mu n} \approx 0 \tag{46}$$

For pions, the nuclear reactions to consider are of the type $\pi + A \rightarrow x$. However, elastic scattering can be ignored here because the energies of interest are high enough so that elastic scattering has little net effect. The net result of elastic scattering at high energies is a small change in direction and energy which are negligible in practical calculations. At low energies, energy loss due to nuclear collisions can become important and are included in a nuclear stopping power in HZETRN [2, 7]. For the analysis here, pions and muons have very low flux at low energies, so nuclear stopping should have little effect and will not be discussed further. Only inelastic pion nucleus reactions will be considered. Inelastic cross sections scale approximately with the size of the nucleus, so they will be approximately proportional to the cross section for pion-proton cross sections,

where the constant of proportionality is approximately equal to the ratios of the areas. The cross-sectional area of a nucleus is area $\approx A^{\frac{2}{3}}$ where A is the atomic number of the target nuclei and the cross section is in units of barns. So,

$$\sigma_{\pi A} \approx A^{0.73}\sigma_{\pi p} \tag{47}$$

where the more accurate value of 0.73 determined by Ranft [3] was used instead of two-thirds. Parameterizations of microscopic pion-proton cross sections are given in the following sections.

4.1.1 Parameterizations for Reaction $\pi^{\pm}p \to x$

Data for inelastic cross sections were scarce, so elastic cross section data from Groom et al. [17] were subtracted from the above parameterization for the total cross section to get more inelastic data. These "calculated inelastic data" were used in conjunction with experimental data [18, 19] to parameterize the inelastic cross section.

The total cross section (elastic + inelastic) parameterization is split into two separate parts. One part is for low pion momentum, and the other is for high momentum and was taken from Groom et al. [17]. The data used were also taken from Groom et al. [17].

4.1.2 Parameterization for Reaction $\pi^{+}p \to x$

For $P_{\text{lab}} < 54.89$ (GeV/c):

$$\sigma_t^{\pi^+ + p} = \frac{a_1}{1+[(x-a_2)/a_3]^2} + \frac{a_4}{1+[(x-a_5)/a_6]^2} + a_7\left[0.5 + \frac{atan(\frac{x-a_8}{a_9})}{\pi}\right] + a_{10} \tag{48}$$

where $x = \log(P_{\text{lab}})$. Also, $a_1 = 4.422$, $a_2 = -1.267$, $a_3 = 0.6411$, $a_4 = 1.915$, $a_5 = 0.3248$, $a_6 = 0.4602$, $a_7 = 2.567$, $a_8 = 0.7135$, $a_9 = 0.3089$, and $a_{10} = 0.5633$.

For $P_{\text{lab}} > 54.89$ (GeV/c):

$$\sigma_t^{\pi^\pm + p} = X s^{\epsilon} + Y_1 s^{-\eta_1} - Y_2 s^{-\eta_2} \tag{49}$$

where $s = m_\pi^2 + m_p^2 + 2.0 m_p \sqrt{p_{\text{lab}}^2 + m_\pi^2}$ is the invariant mass squared, and where $X = 11.883$, $Y_1 = 28.59$, $Y_2 = 5.90$, $\epsilon = 0.093$, $\eta_1 = 0.358$, and $\eta_2 = 0.560$ [17]. A comparison of this parameterization with data [17] is given in figure 3.

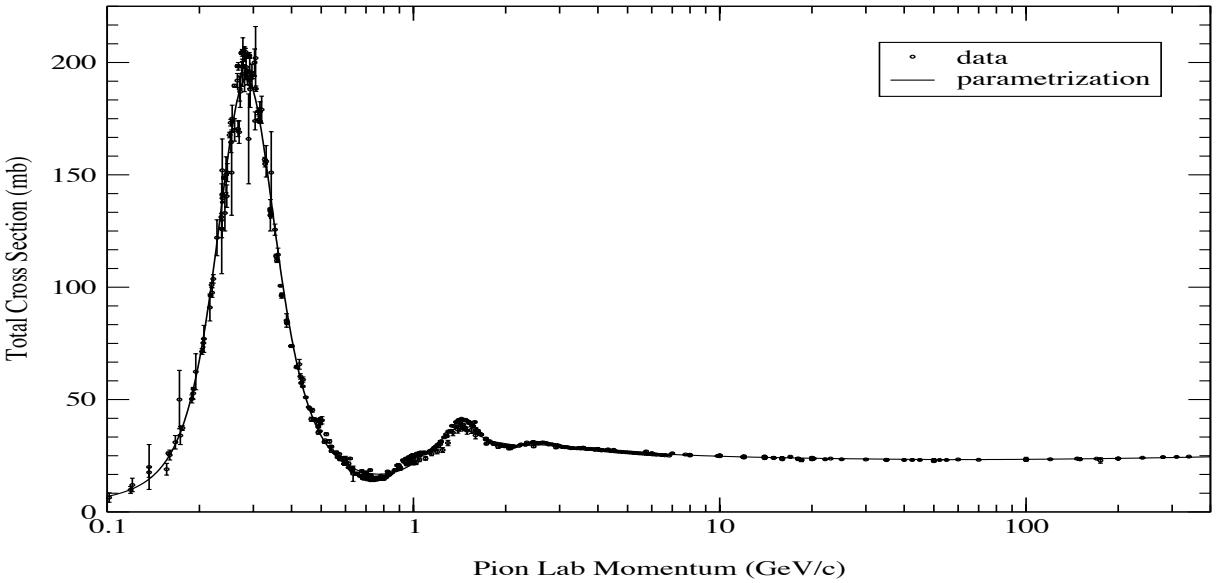

Figure 3: Data [17] plotted against the parameterization given by equations (48) and (49) for total cross section for the reaction $\pi^+ + p \to x$.

4.1.3 Parameterization for Reaction $\pi^- p \to x$

For $P_{\rm lab} < 1.283$ (GeV/c):

$$\begin{aligned}\sigma_t^{\pi^-+p} &= b_1 \exp\Big[-0.5\big(\frac{x-b_2}{b_3}\big)^2\Big] + b_4 \exp\Big[-0.5\big(\frac{x-b_5}{b_6}\big)^2\Big] \\ &\quad + b_7\Big[1+\big(\frac{x}{b_8}\big)^2\Big]^{-1} + b_9\Big[1+\big(\frac{x-b_{10}}{b_{11}}\big)^2\Big]^{-1}\end{aligned} \tag{50}$$

where $x = \log P_{\rm lab}$, $b_1 = 48.3$, $b_2 = -1.27$, $b_3 = 0.182$, $b_4 = 33.9$, $b_5 = 0.903$, $b_6 = 2.04$, $b_7 = 26.8$, $b_8 = 0.0782$, $b_9 = 16.5$, $b_{10} = -0.333$, and $b_{11} = 0.0744$.

For $P_{\rm lab} > 1.283$ (GeV/c):

$$\sigma_t^{\pi^\pm+p} = X s^{\epsilon} + Y_1 s^{-\eta_1} + Y_2 s^{-\eta_2} \tag{51}$$

where $s = m_\pi^2 + m_p^2 + 2.0 m_p \sqrt{p_{\rm lab}^2 + m_\pi^2}$ is the invariant mass squared, and where $X = 11.883$, $Y_1 = 28.59$, $Y_2 = 5.90$, $\epsilon = 0.093$, $\eta_1 = 0.358$, and $\eta_2 = 0.560$ [17]. A comparison of the parameterization for the reaction $\pi^- + p \to x$ with data [17] is given in figure 4.

4.1.4 Parameterizations for Reaction $\pi^\pm p \to x$: Inelastic

$$\sigma_{\rm inelastic}^{\pi^++p} = \frac{c_1}{1+\exp[(c_2-x)/c_3]} + c_4 \exp\Big[-0.5\big(\frac{x-c_5}{c_6}\big)\Big] \tag{52}$$

where $x = \log(P_{\rm lab})$, $c_1 = 20.1$, $c_2 = 0.0653$, $c_3 = 0.197$, $c_4 = 1.18$, $c_5 = 0.869$, and $c_6 = 0.705$.

The low energy parameterization for the inelastic cross section for negative pions on

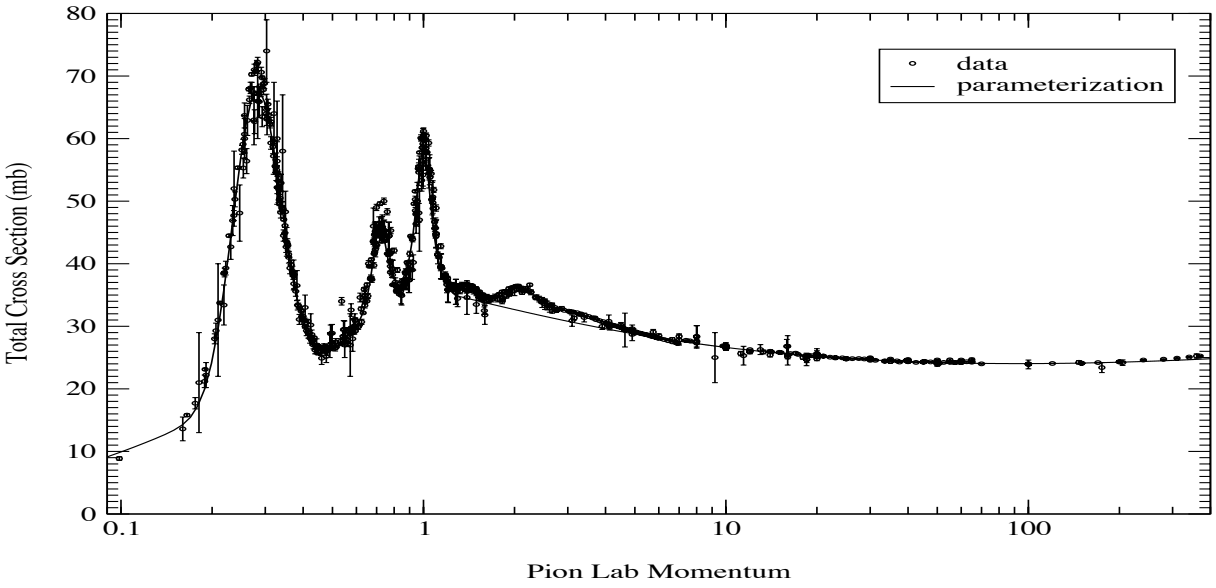

Figure 4: Data [17] plotted against parameterization given by equations (50) and (51) for total cross section for reaction $\pi^- + p \to x$.

protons is the same form as that for the total cross section, but the parameters are different.

For $P_{\rm lab} < 5.02234$ (GeV/c):

$$\begin{aligned}\sigma^{\pi^-+p}_{\rm inelastic} &= d_1 \exp\Big[-0.5\big(\frac{x-d_2}{d_3}\big)^2\Big] + d_4 \exp\Big[-0.5\big(\frac{x-d_5}{d_6}\big)^2\Big] \\ &\quad + d_7\Big[1+\big(\frac{x}{d_8}\big)^2\Big]^{-1} + d_9\Big[1+\big(\frac{x-d_{10}}{d_{11}}\big)^2\Big]^{-1}\end{aligned} \tag{53}$$

where $x = \log P_{\rm lab}$, $d_1 = 31.9$, $d_2 = -1.27$, $d_3 = 0.176$, $d_4 = 23.5$, $d_5 = 1.32$, $d_6 = 2.37$, $d_7 = 12.5$, $d_8 = 0.0745$, $d_9 = 7.73$, $d_{10} = -0.33$, and $d_{11} = 0.0518$.

For $P_{\rm lab} > 5.02234$ (GeV/c):

$$\sigma^{\pi^-+p}_{\rm inelastic} = X s^{\epsilon} + Y_1 s^{-\eta_1} + Y_2 s^{-\eta_2} \tag{54}$$

where $s = m_\pi^2 + m_p^2 + 2.0 m_p \sqrt{p_{\text{lab}}^2 + m_\pi^2}$ is the invariant mass squared, and where $X = 10.9$, $Y_1 = 20.2$, $Y_2 = 3.9$, $\epsilon = 0.093$, $\eta_1 = 0.358$, and $\eta_2 = 0.560$. These parameterizations are shown in figures 5 and 6.

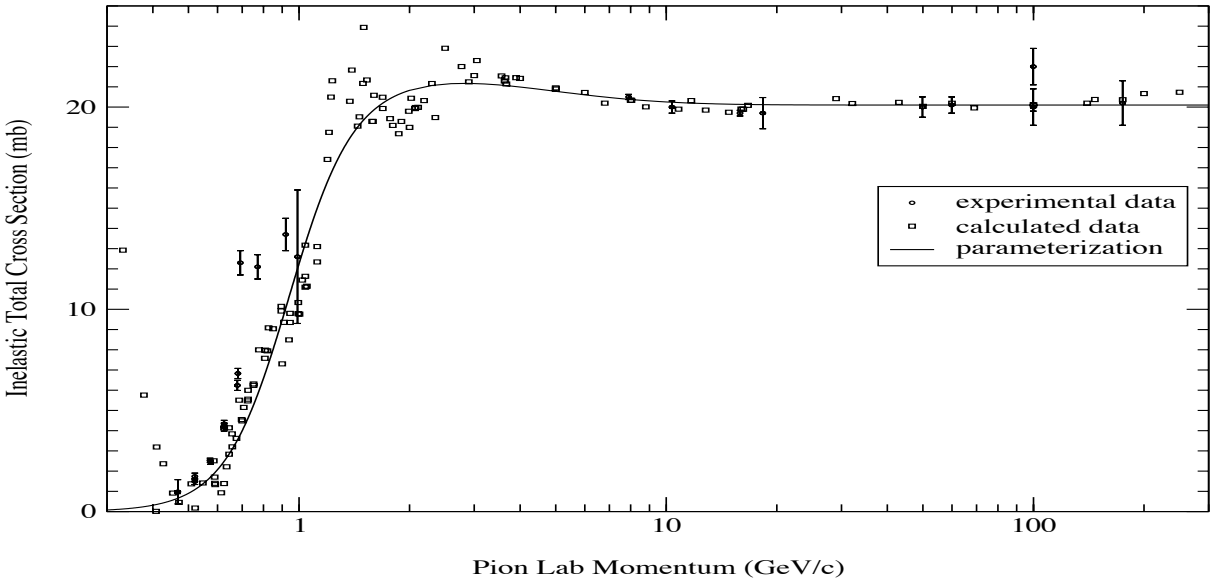

Figure 5: Parameterization given by equation (52) for total inelastic cross section for π^+-proton collisions is plotted against experimental data [18] and "calculated data." The latter was calculated by subtracting elastic cross section data [17] from total cross section parameterization given by equations (48) and (49).

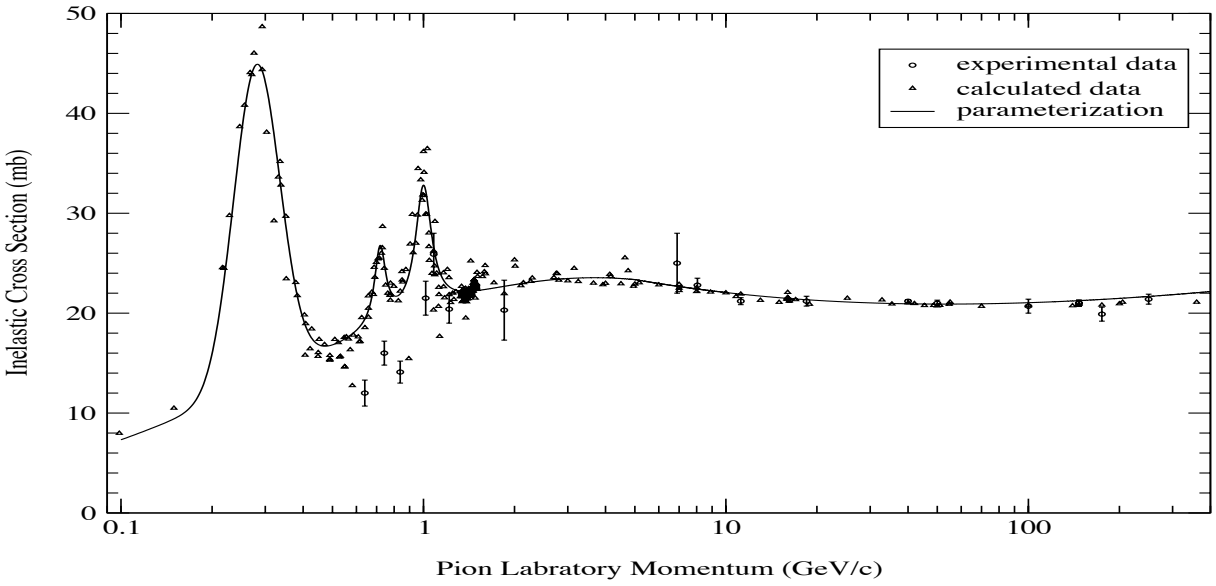

Figure 6: Parameterization given by equations (53) and (54) for total inelastic cross section for π^--proton collisions is plotted against experimental data [19] and "calculated data." The latter was calculated by subtracting elastic cross section data [17] from the total cross section parameterization given by equations (50) and (51).

4.2 Stopping Power

The Bethe theory shows that stopping power of other particles can be approximately scaled from the stopping power of protons [2, 7]

$$\tilde{S}_j = \frac{Z_j^2}{A_j} \tilde{S}_p \tag{55}$$

where Z is charge and A is mass in amu. The function $\tilde{S}_p$ is calculated in HZETRN and is an input to the pion transport subroutine. Because a positive pion has the same charge as a proton, it seems reasonable that the electromagnetic interactions could be simply scaled

from that of a proton to correct for the difference in mass. There might be some concern whether this is the case for negative pions and muons as well. The difference between stopping powers of protons and anti-protons was measured by Agnello et al. [20]. There seems to be an important difference at low energies up to the order of 100 KeV. Pions and muons are expected to have a similar behavior, and these energies are below where any significant pion or muon flux is expected. See [2, 7, 8, 9, 10, 11] for a more detailed description of stopping power.

4.3 Production Cross Sections

Cross sections a_{jk} contain particle production caused by collisions with the medium and the decay of unstable particles. For collisions, the macroscopic production cross section $a_{jk}(r, r')$ can be expressed in terms of microscopic spectral distributions and densities of target nuclei. In the straight ahead approximation, there is no angular dependence. Thus,

$$\begin{aligned} a_{jk}(r, r') &= \sigma_{jk}(r, r') + u_{jk}(r, r') \\ &= \sum_{n=1}^{N} \rho_n \frac{d\sigma_{jnk}}{dE}(r, r') + u_{jk}(r, r') \end{aligned} \tag{56}$$

where $u_{jk}(r, r')\phi_k(r')$ is the number of particles per unit volume of type k with energy E' decaying into a particle of type j and energy E. The reaction being considered is of the type $k \to j + x$, where x represents any combination of particles and $\sigma_{jnk}(r, r')$ is the microscopic cross section for a particle of type k with energy $E'(r')$ colliding with a nuclei of type n and producing a particle of type j and energy $E(r)$. There are many different

ways that this production can happen, so the appropriate cross sections are inclusive. In other words, the reaction is of the type $k + n \rightarrow j + x$, where x represents any allowed combination of particles. The rest frame of the medium is assumed, so that the nuclei are stationary when neglecting thermal motion. Thermal motion can safely be neglected when the energies of the radiations are much greater than the thermal energy of the medium.

4.3.1 Production From Decay

The term $u_{jk}(r, r')$ takes into account all possible decay channels. In other words,

$$u_{jk}(r, r') = \sum_i u^i_{jk}(r, r') \tag{57}$$

The index i indicates a particular reaction of the type $a \rightarrow b + c + d$, in which a, b, c, and d are all individual particles.

As a first approximation, one can assume that muons are created entirely from the decay of charged pions. Charged pions decay by the reaction $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$ with a branching ratio of 99.99 percent. Because this is a two-body decay, the macroscopic particle production cross section can be calculated exactly by using only kinematic constraints, as shown in appendix C and in Gaisser [21].

$$u_{\mu\pi}(E_{\mu}, E_{\pi}) = \frac{1}{\beta_{\pi}\gamma_{\pi}\tau_{\pi}} \frac{1}{2\gamma_{\pi}\beta_{\pi}p^*} \tag{58}$$

where $p^* = \sqrt{(E^*)^2 - m_{\mu}^2}$ and $E^* = \frac{m_{\pi}^2 + m_{\mu}^2 - m_{\nu}}{2m_{\pi}}$ and where $m_{\nu} \approx 0$ is the mass of the neutrino. Note that the cross section $u_{\mu\pi}(E_{\mu}, E_{\pi})$ has the above form only in the energy

region allowed by conservation of energy and momentum as specified in appendix C.

Production of pions from the decay of other particles (kaons etc.) is presently neglected.

4.3.2 Production From Collisions

$$\sigma_{jk}(r, r') = \sum_m \rho_m \Big(\frac{d\sigma}{dE}\Big)_{k+m \to j+x}(r, r') \tag{59}$$

Presently, pions are assumed to be produced solely by the collisions,

$$A + A' \rightarrow \pi^{\pm} + x \tag{60}$$

$$\pi^+ + A \rightarrow \pi^+ + x \tag{61}$$

$$\pi^- + A \rightarrow \pi^+ + x \tag{62}$$

$$\pi^+ + A \rightarrow \pi^- + x \tag{63}$$

$$\pi^- + A \rightarrow \pi^- + x \tag{64}$$

where A represents a nucleus of atomic number A. Protons and neutrons are presently being treated equivalently with A set to unity.

Parameterizations of inclusive pion production in proton-proton collisions are given in Blattnig et al. [4]. All pion production cross sections are based on a combination of these parameterizations and parameterizations from Ranft [3]. These parameterizations assume energy in GeV while HZETRN has energy in MeV/amu. For the reaction $A+A' \rightarrow \pi^{\pm}+x$,

$$\Big(\frac{d\sigma}{dE}\Big)_{A+A' \to \pi+x} = A^{0.69} A'^{0.69} \Big(\frac{d\sigma}{dE}\Big)_{p+p \to \pi+x} \tag{65}$$

For reactions of type $\pi + A \rightarrow \pi + x$, the pion projectile is treated like a proton rescaled by the inelastic cross sections.

$$\left(\frac{d\sigma}{dE}\right)_{\pi+A\rightarrow\pi+x} = \frac{\sigma_{\pi A}^{\text{inelastic}}}{\sigma_{pA}^{\text{inelastic}}} A^{0.73} \left(\frac{d\sigma}{dE}\right)_{p+p\rightarrow\pi+x} \tag{66}$$

$$= 0.62 A^{0.77} \left(\frac{d\sigma}{dE}\right)_{p+p\rightarrow\pi+x} \tag{67}$$

where parameterizations from Ranft [3] were used for the inelastic cross sections. Note that since the reaction $p + p \rightarrow \pi + x$ does not contain an elastic channel, the cross section for $\pi^{\pm} + A \rightarrow \pi^{\pm} + x$ will not include the elastic part. Since elastic reactions are also neglected as losses, elastic scattering is completely neglected in the present approximation. As discussed in section 4.1, elastic scattering has little net effect for the energies considered here. The following are parameterizations of charged pion production from proton-proton collisions from Blattnig et al. [4]. The symbols E_p and E_π are the proton and pion kinetic energies in the lab frame.

The positively charged pion spectral distribution for the range 0.3-2 GeV is represented by the following equations:

$$\begin{aligned} F_2 &= C_1 E_\pi^{C_2} + C_3 E_p^{C_4} \\ F_1 &= \exp\left(C_5 + \frac{C_6}{\sqrt{E_p}} + C_7 E_p^{C_8} + C_9 E_\pi^{C_{10}} + C_{11} E_\pi^{C_{12}} E_p^{C_{13}} + C_{14} \ln E_p\right) \\ \left(\frac{d\sigma}{dE}\right)_{\text{lab}} &= C_{15} E_\pi^{C_{16}} \frac{F_1}{F_2} + C_{17} E_\pi^{C_{18}} \exp\left(C_{19}\sqrt{E_\pi} + C_{20}\sqrt{E_p}\right) \end{aligned} \tag{68}$$

with constants C_i given in table 1.

The positively charged pion spectral distribution for the range 2-50 GeV is represented

$C_1 = 2.2 \times 10^{-8}$	$C_8 = -1.75$	$C_{15} = 2.5 \times 10^6$
$C_2 = -2.7$	$C_9 = -29.4$	$C_{16} = 0.25$
$C_3 = 4.22 \times 10^{-7}$	$C_{10} = 0.0938$	$C_{17} = 976$
$C_4 = -1.88$	$C_{11} = -24.4$	$C_{18} = 2.3$
$C_5 = 22.3$	$C_{12} = 0.0312$	$C_{19} = -46$
$C_6 = 1.98$	$C_{13} = 0.0389$	$C_{20} = -0.989$
$C_7 = -0.28$	$C_{14} = 1.78$	

Table 1: Constants for Equation (68)

$D_1 = 4.5 \times 10^{-11}$	$D_7 = -35.3$	$D_{13} = 60322$
$D_2 = -2.98$	$D_8 = 0.0938$	$D_{14} = 1.18$
$D_3 = 1.18 \times 10^{-9}$	$D_9 = -22.5$	$D_{15} = -72.2$
$D_4 = -2.55$	$D_{10} = 0.0313$	$D_{16} = 0.941$
$D_5 = 22.3$	$D_{11} = 2.5 \times 10^6$	$D_{17} = 0.1$
$D_6 = -0.765$	$D_{12} = 0.25$	

Table 2: Constants for Equation (69)

by the following equations:

$$\begin{aligned} F_2 &= D_1 E_\pi^{D_2} + D_3 E_p^{D_4} \\ F_1 &= \exp\left(D_5 + \frac{D_6}{\sqrt{E}_p} + D_7 E_\pi^{D_8} + D_9 E_\pi^{D_{10}}\right) \\ \left(\frac{d\sigma}{dE}\right)_{\text{lab}} &= D_{11} E_\pi^{D_{12}} \frac{F_1}{F_2} + D_{13} E_\pi^{D_{14}} \exp\left(D_{15}\sqrt{E_\pi} + D_{16} E_p^{D_{17}}\right) \end{aligned} \tag{69}$$

with constants D_i given in table 2.

The negatively charged pion spectral distribution for the range 0.3-2 GeV is represented by the following equations:

$$\begin{aligned} F_2 &= G_1 E_\pi^{G_2} + G_3 E_p^{G_4} \\ F_1 &= \exp\left(G_5 + \frac{G_6}{\sqrt{E}_p} + G_7 E_\pi^{G_8} + G_9 E_\pi^{G_{10}}\right) \\ \left(\frac{d\sigma}{dE}\right)_{\text{lab}} &= E_\pi^{G_{11}} \left[G_{12} \frac{F_1}{F_2} + G_{13} \exp(G_{14}\sqrt{E}_\pi)\right] \end{aligned} \tag{70}$$

with constants G_i given in table 3.

The negatively charged pion spectral distribution for the range 2-50 GeV is represented

$G_1 = 1.06 \times 10^{-9}$	$G_6 = -1.5$	$G_{11} = 0.25$
$G_2 = -2.8$	$G_7 = -30.5$	$G_{12} = 2.5 \times 10^6$
$G_3 = 3.7 \times 10^{-8}$	$G_8 = 0.0938$	$G_{13} = 7.96$
$G_4 = -1.89$	$G_9 = -24.6$	$G_{14} = -49.5$
$G_5 = 22.3$	$G_{10} = 0.0313$	

Table 3: Constants for Equation (70)

$H_1 = 2.39 \times 10^{-10}$	$H_7 = -31.3$	$H_{13} = 60322$
$H_2 = -2.8$	$H_8 = 0.0938$	$H_{14} = 1.1$
$H_3 = 1.14 \times 10^{-8}$	$H_9 = -24.9$	$H_{15} = -65.9$
$H_4 = -2.3$	$H_{10} = 0.0313$	$H_{16} = -9.39$
$H_5 = 22.3$	$H_{11} = 2.5 \times 10^6$	$H_{17} = -1.25$
$H_6 = -2.23$	$H_{12} = 0.25$	

Table 4: Constants for Equation (71)

by the following equations:

$$\begin{aligned} F_2 &= H_1 E_\pi^{H_2} + H_3 E_p^{H_4} \\ F_1 &= \exp\left(H_5 + \frac{H_6}{\sqrt{E_p}} + H_7 E_\pi^{H_8} + H_9 E_\pi^{H_{10}}\right) \\ \left(\frac{d\sigma}{dE}\right)_{\text{lab}} &= H_{11} E_\pi^{H_{12}} \frac{F_1}{F_2} + H_{13} E_\pi^{H_{14}} \exp\left(H_{15}\sqrt{E_\pi} + H_{16} E_p^{H_{17}}\right) \end{aligned} \tag{71}$$

with constants H_i given in table 4.

5 Results

By using the solution to the transport equation given in section 3.2 and the database given in section 4, charged pion and muon fluxes were calculated after the initial cosmic ray fluxes were transported through various depths of aluminum and of water. Aluminum is commonly used in the construction of spacecraft, and water is a very large component of human tissue. The initial fluxes were given by the model for the 1977 solar minimum described in Wilson et al. [6]. The results of this calculation are given by figures 7-16.

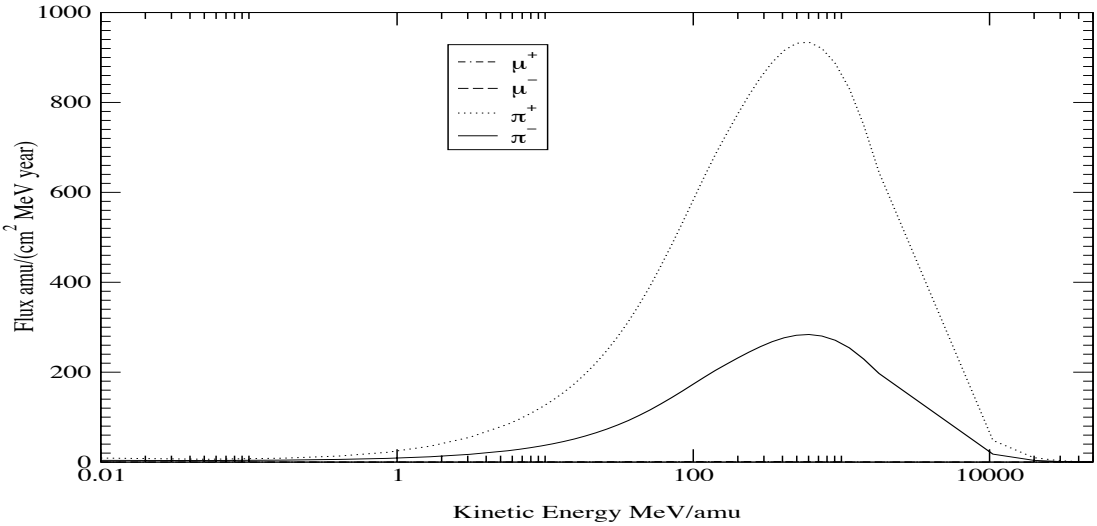

Figure 7: Pion and muon fluxes after transport through 5 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum for 1977 solar minimum.

Figure 7 shows pion and muon fluxes after transport through $5\frac{\text{gm}}{\text{cm}^2}$ of aluminum. The muon fluxes are too small to be seen. Figure 8 is the same as figure 7 except that it shows pion and muon fluxes as a percentage of the total calculated radiation flux. The percentages are large for higher energies. The peak in the percentages is at a higher energy than the peak in the actual fluxes. The peak in the percentage is at an energy at which the pion fluxes are relatively small. These large percentages at high energy are due to the falling off of the ion fluxes at high energy. Most of the radiation is at lower energies where the pion fluxes are relatively small.

Figures 9 and 10, and figures 11 and 12 are the same as figures 7 and 8 except that transport through $10\frac{\text{gm}}{\text{cm}^2}$ and $20\frac{\text{gm}}{\text{cm}^2}$ of aluminum is shown. Figures 13 and 14 show pion and muon fluxes after transport through $20\frac{\text{gm}}{\text{cm}^2}$ of aluminum and $5\frac{\text{gm}}{\text{cm}^2}$ of water. Water is

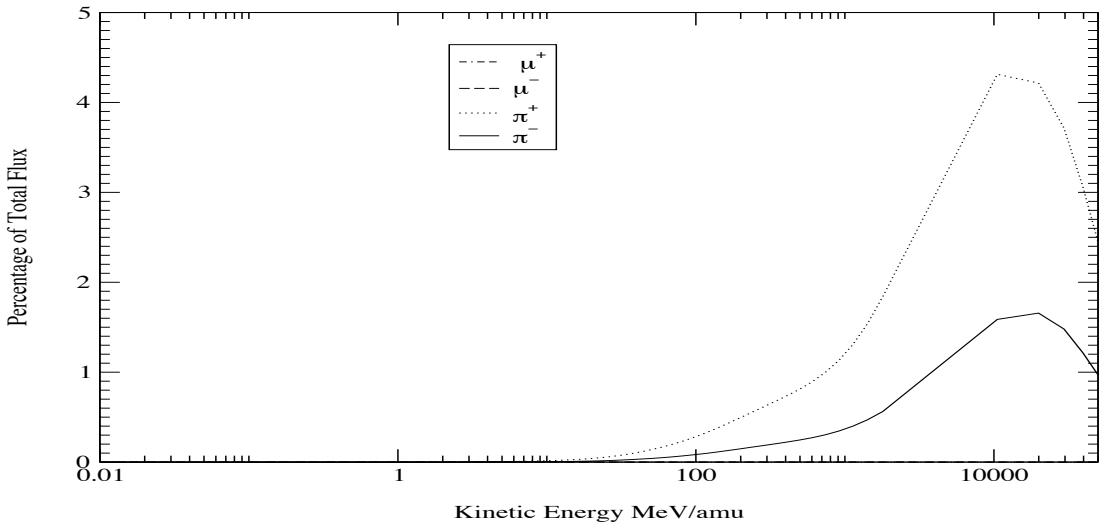

Figure 8: Percentage of total radiation flux due to pions and muons after transport through 5 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum for 1977 solar minimum.

used to simulate tissue, and $5\frac{\text{gm}}{\text{cm}^2}$ is the approximate depth of the blood forming organs in a person [2]. Figure 15 shows the sum of the charged pion and muon fluxes at various depths in the aluminum and water. These fluxes steadily increase as more matter is traversed. The increase is due to an accumulation of pions as more nuclear collisions occur. Muons are negligible for these small amounts of material. Figure 16 is the same as figure 15 except that the percentage of the total radiation flux is shown.

Muon fluxes are negligible compared to the pion fluxes, and the fluxes were also graphed separately in figure 17 so that the shape of the distributions could be seen. The fluxes are so low simply because there is not enough time for a significant amount of pions to decay. Muons might be much more important for thicker shielding such as the martian atmosphere.

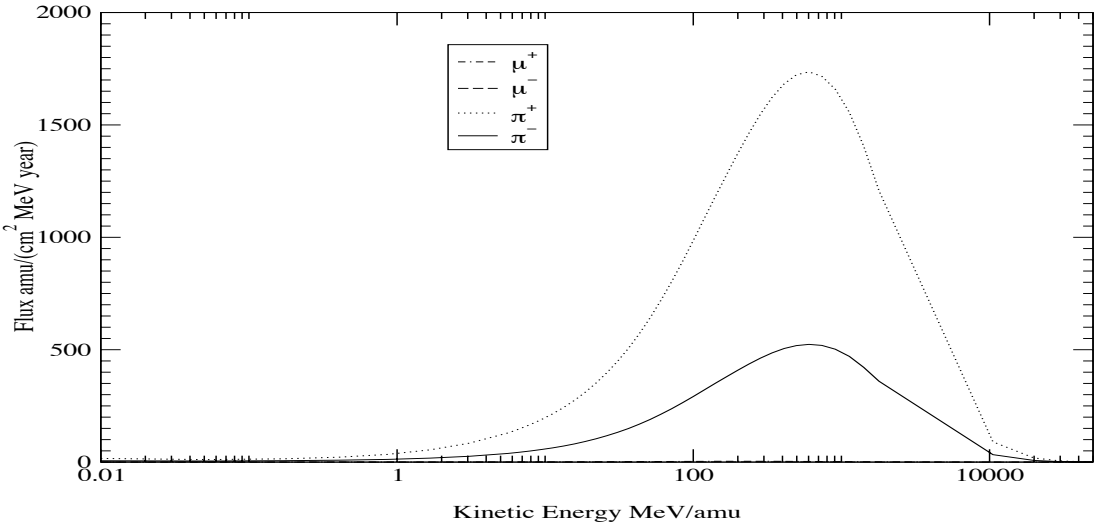

Figure 9: Pion and muon fluxes after transport through 10 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum for 1977 solar minimum.

In summary, a one-dimensional deterministic muon and pion transport code was developed. This code is capable of transporting pions and muons over arbitrary distances for a wide variety of materials when it is coupled to the ion and neutron transport code HZETRN. All the needed cross sections were either derived or taken from other sources. Example results for transport through aluminum and water were then presented and shown to be reasonable. The major shortcomings of this calculation are as follows:

1. The nuclear fragmentation database does take the effects of pion production into account. Because pion production is treated independently from other fragments, the total energy from all fragments in a collision will not necessarily add up to the energy of the initial colliding particles, so energy will not be strictly conserved.

2. The parameterizations of cross sections describing pion production were based

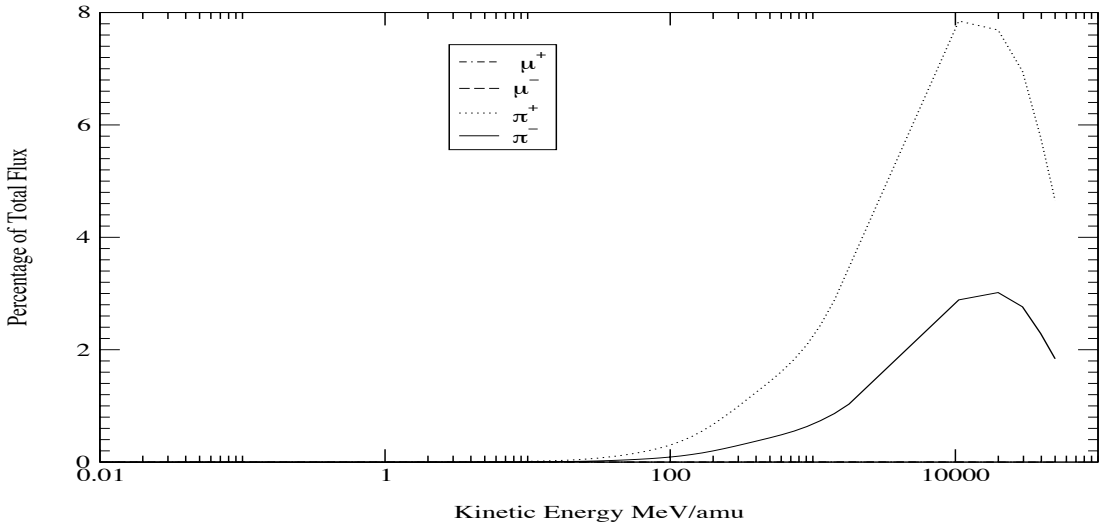

Figure 10: Percentage of total radiation flux due to pions and muons after transport through 10 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum for 1977 solar minimum.

mainly on high energy data. The physical mechanisms for pion production are different at low energy, so the cross sections based on high energy data will be somewhat inaccurate at low energy. Improvements in these cross sections are presently being developed by using a meson exchange model where pions are produced from baryon resonances. Further work improving this code is presently underway.

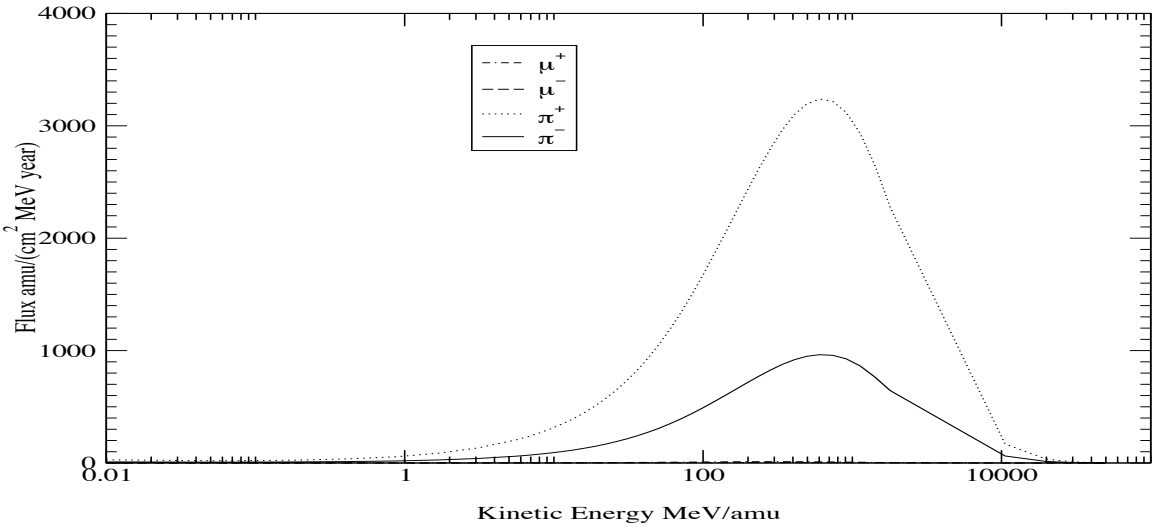

Figure 11: Pion and muon fluxes after transport through 20 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum for 1977 solar minimum.

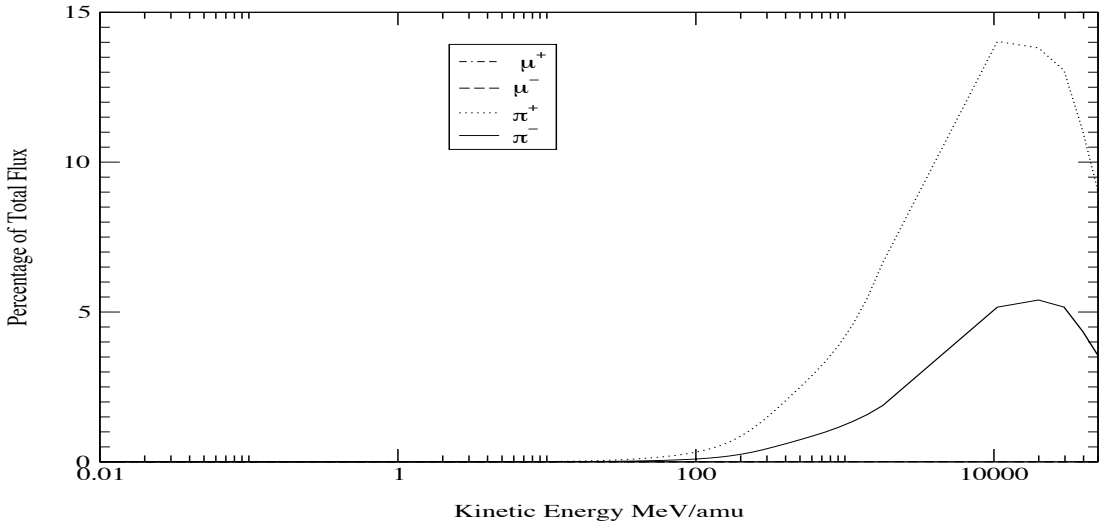

Figure 12: Percentage of total radiation flux due to pions and muons after transport through 20 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum for 1977 solar minimum.

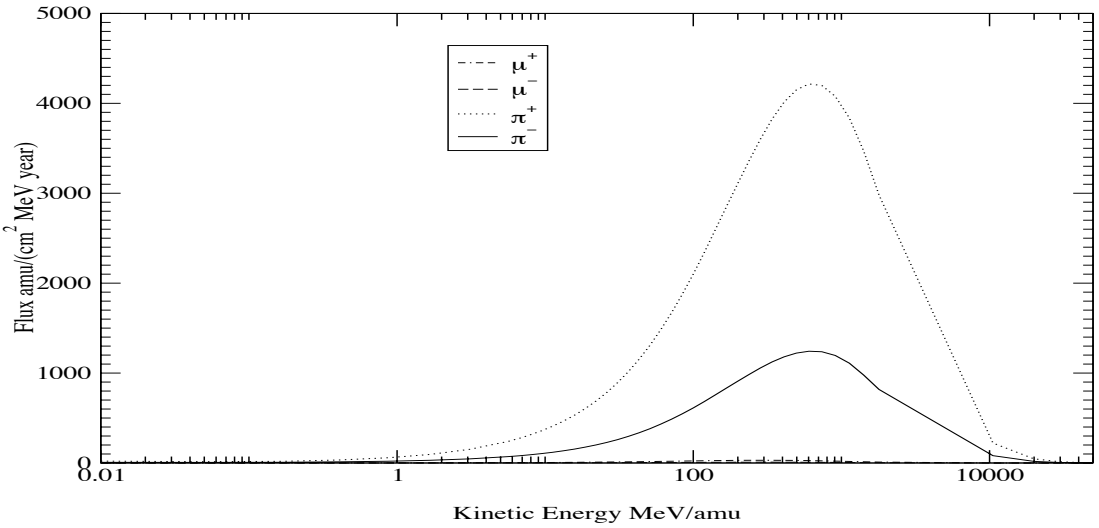

Figure 13: Pion and muon fluxes after transport through 20 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum and 5 $\frac{\text{gm}}{\text{cm}^2}$ of water for 1977 solar minimum.

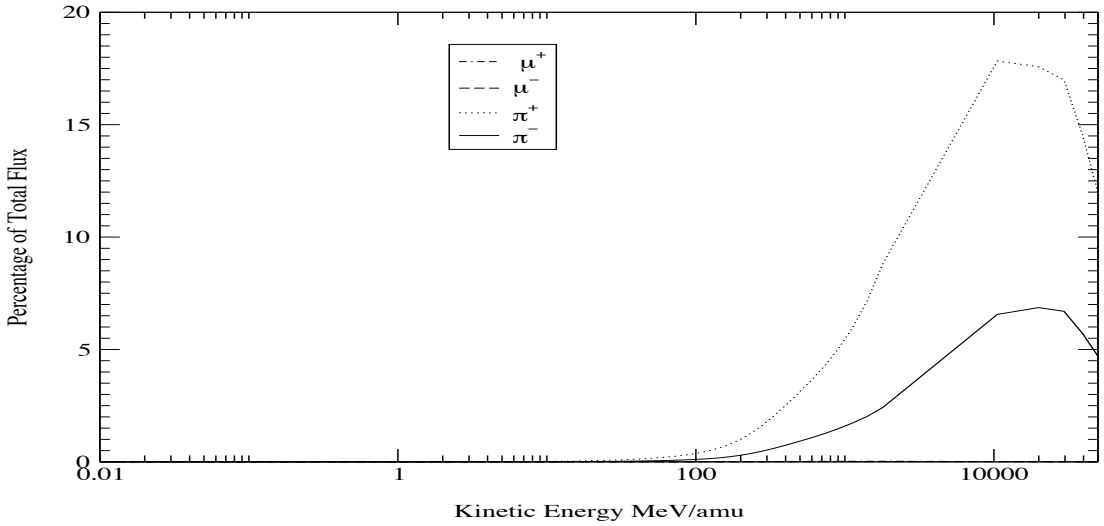

Figure 14: Percentage of total radiation flux due to pions and muons after transport through 20 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum and 5 $\frac{\text{gm}}{\text{cm}^2}$ of water for 1977 solar minimum.

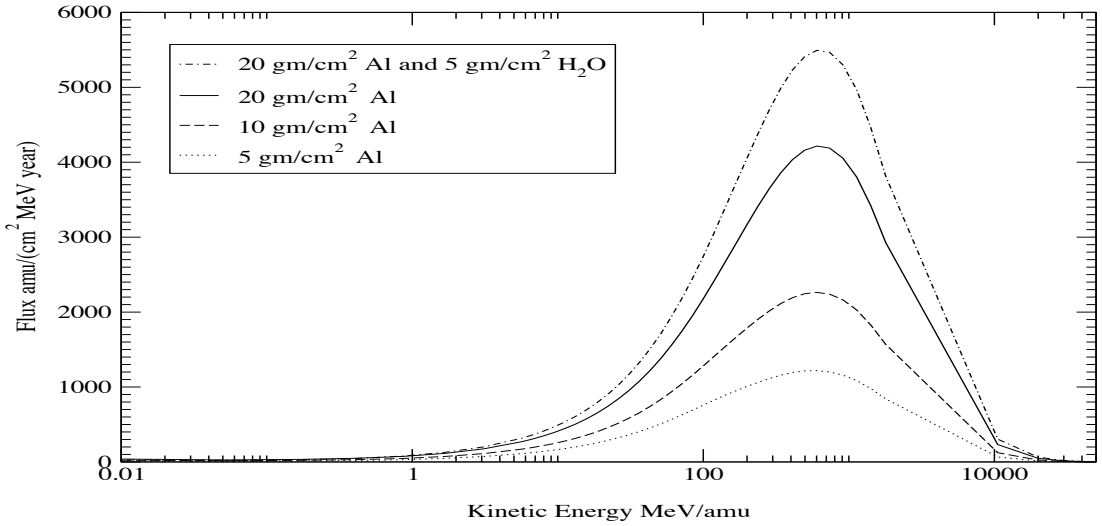

Figure 15: The sum of pion and muon fluxes after transport through various depths of aluminum and of water for 1977 solar minimum.

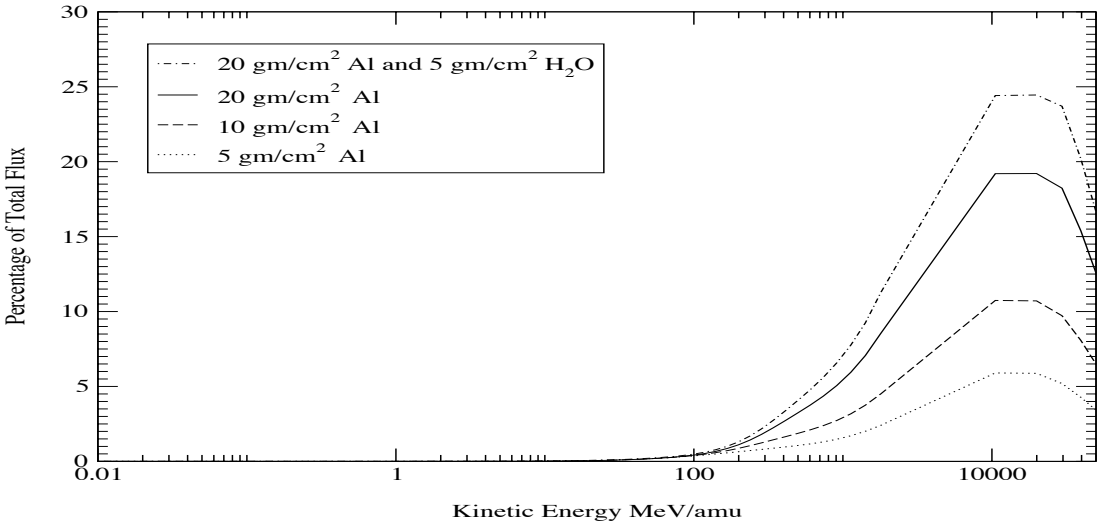

Figure 16: Percentage of the total flux due to the sum of pion and muon fluxes, after transport through various depths of aluminum and of water for 1977 solar minimum.

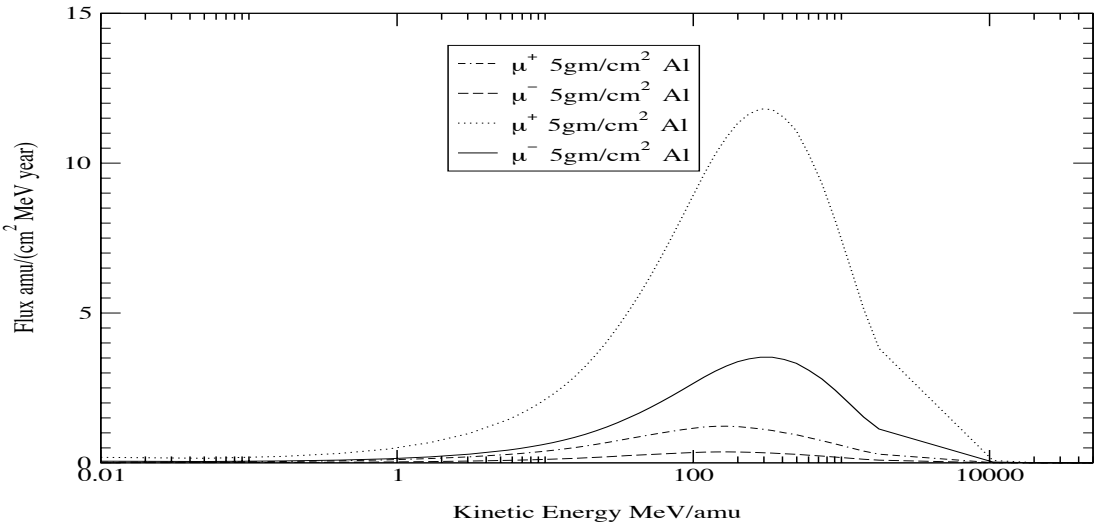

Figure 17: Muon fluxes after transport through 5 $\frac{\text{gm}}{\text{cm}^2}$ and 20 $\frac{\text{gm}}{\text{cm}^2}$ of aluminum for the 1977 solar minimum.

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A The Solution of the Transport Equation

As shown in section 2 and in equation (22), the continuous slowing down and straight-ahead approximations can reduce the transport equation (6) to the following equation:

$$\Big[\frac{\partial}{\partial x} - \frac{\partial}{\partial r_j} + a_j(r_j)\Big]\psi_j(x, r_j) = q_j(x, r_j) \tag{72}$$

where

$$q_j(x, r_j) = S_j(r_j) \sum_k \int dr_k a_{jk}(r_j, r_k)\psi_k(x, r_k) \tag{73}$$

represents the creation of particles of type j, at position x, with range r_j. The method of inverting this equation, and approximating the solution that was mentioned in section 3 will now be discussed in detail. A similar analysis can also be seen in Wilson et al. [2] and in Shinn et al. [14].

Equation (22) or (72) cannot be directly integrated because it contains two partial derivatives. If the following variable changes are made (method of characteristics), then the partial derivatives will become an ordinary single derivative, which can be directly integrated [6, 15].

$$\begin{aligned} \eta_j &\equiv x - r_j \\ \xi_j &\equiv x + r_j \end{aligned} \tag{74}$$

therefore,

$$\begin{aligned}\frac{\partial}{\partial x}-\frac{\partial}{\partial r_j} &= \frac{\partial \eta_j}{\partial x}\frac{\partial}{\partial \eta_j}+\frac{\partial \xi_j}{\partial x}\frac{\partial}{\partial \xi_j}-\frac{\partial \eta_j}{\partial r_j}\frac{\partial}{\partial \eta_j}-\frac{\partial \xi_j}{\partial r_j}\frac{\partial}{\partial \xi_j} \\ &= \frac{\partial}{\partial \eta_j}+\frac{\partial}{\partial \xi_j}+\frac{\partial}{\partial \eta_j}-\frac{\partial}{\partial \xi_j} \\ &= 2\frac{\partial}{\partial \eta_j} \end{aligned} \tag{75}$$

Substituting equations (74) and (75) into equation (22) or (72) results in

$$\Big[2\frac{\partial}{\partial \eta_j}+a_j\big(\frac{\xi_j-\eta_j}{2}\big)\Big]\psi_j\big(\frac{\xi_j+\eta_j}{2},\frac{\xi_j-\eta_j}{2}\big)=q_j\big(\frac{\xi_j+\eta_j}{2},\frac{\xi_j-\eta_j}{2}\big) \tag{76}$$

This equation can now be integrated by introducing the following integrating factor [2, 6, 14],

$$\mu_j(\xi_j,\eta_j)=\exp\Big[\frac{1}{2}\int_{-\xi_j}^{\eta_j}d\eta' a_j\big(\frac{\xi_j-\eta'}{2}\big)\Big] \tag{77}$$

which implies

$$\begin{aligned} 2\frac{\partial}{\partial \eta_j}\mu_j(\xi_j,\eta_j) &= \mu_j(\xi_j,\eta_j)\frac{\partial}{\partial \eta_j}\int_{-\xi_j}^{\eta_j}d\eta' a_j\big(\frac{\xi_j-\eta'}{2}\big) \\ &= \mu_j(\xi_j,\eta_j)a_j\big(\frac{\xi_j-\eta_j}{2}\big) \end{aligned} \tag{78}$$

therefore [15]

$$2\frac{\partial}{\partial \eta_j}\Big[\psi_j\big(\frac{\xi_j+\eta_j}{2},\frac{\xi_j-\eta_j}{2}\big)\mu_j(\xi_j,\eta_j)\Big]=\mu_j(\xi_j,\eta_j)q_j\big(\frac{\xi_j+\eta_j}{2},\frac{\xi_j-\eta_j}{2}\big) \tag{79}$$

A simple integral equation can now be formed by using equation (79) in equation (76) and integrating over η_j' from $-\xi_j$ to η_j,

$$\begin{aligned} &2\int_{-\xi_j}^{\eta_j}\frac{\partial}{\partial \eta_j'}\Big[\psi_j\big(\frac{\xi_j+\eta_j'}{2},\frac{\xi_j-\eta_j'}{2}\big)\mu_j(\xi_j,\eta_j')\Big]d\eta_j' \\ &\quad=\int_{-\xi_j}^{\eta_j}d\eta_j' q_j\big(\frac{\xi_j+\eta_j'}{2},\frac{\xi_j-\eta_j'}{2}\big)\mu_j(\xi_j,\eta_j') \end{aligned} \tag{80}$$

Through further simplification and the identification $\mu_j(\xi_j, -\xi_j) = 1$, the following result can be obtained:

$$2\Big[\psi_j\big(\frac{\xi_j+\eta_j}{2}, \frac{\xi_j-\eta_j}{2}\big)\mu_j(\xi_j,\eta_j) - \psi_j(0,\xi_j)\mu(-\xi_j,-\xi_j)\Big] = \int_{-\xi_j}^{\eta_j} d\eta_j' \mu_j(\xi_j,\eta_j') q_j\big(\frac{\xi_j+\eta_j'}{2}, \frac{\xi_j-\eta_j'}{2}\big)$$

giving [15]

$$\psi_j\big(\frac{\xi_j+\eta_j}{2}, \frac{\xi_j-\eta_j}{2}\big) = \frac{1}{2\mu_j(\xi_j,\eta_j)}\Big[2\psi_j(0,\xi_j) + \int_{-\xi_j}^{\eta_j} d\eta_j' \mu_j(\xi_j,\eta_j') q_j\big(\frac{\xi_j+\eta_j'}{2}, \frac{\xi_j-\eta_j'}{2}\big)\Big] \tag{81}$$

Changing variables back to x and r by inverting equations (74), explicitly writing out μ and simplifying notation by using

$$\begin{aligned} a_j(\eta_j'') &\equiv a_j\big(\frac{x+r_j-\eta_j''}{2}\big) \\ q_j(\eta_j') &\equiv q_j\big(\frac{x+r_j+\eta_j'}{2}, \frac{x+r_j-\eta_j'}{2}\big) \end{aligned} \tag{82}$$

yields the following after some further algebra [2, 14],

$$\begin{aligned} \psi_j(x,r_j) &= \exp\Big[-\frac{1}{2}\int_{-x-r_j}^{x-r_j} d\eta_j'' a_j(\eta_j'')\Big]\psi_j(0,x+r_j) \\ &\quad + \frac{1}{2}\int_{-x-r_j}^{x-r_j} d\eta_j' \exp\Big[-\frac{1}{2}\int_{\eta_j'}^{x-r_j} d\eta_j'' a_j(\eta_j'')\Big] q_j(\eta_j') \end{aligned} \tag{83}$$

Writing out equation (83) in full using the definitions given in equation (82) results in the following, which is equation (23):

$$\begin{aligned} \psi_j(x,r_j) &= \exp\Big[-\frac{1}{2}\int_{-x-r_j}^{x-r_j} d\eta_j'' a_j\big(\frac{x+r_j-\eta_j''}{2}\big)\Big]\psi_j(0,x+r_j) \\ &+ \frac{1}{2}\int_{-x-r_j}^{x-r_j} d\eta_j' \exp\Big[-\frac{1}{2}\int_{\eta_j'}^{x-r_j} d\eta_j'' a_j\big(\frac{x+r_j-\eta_j''}{2}\big)\Big] q_j\big(\frac{x+r_j+\eta_j'}{2}, \frac{x+r_j-\eta_j'}{2}\big) \end{aligned} \tag{84}$$

A.1 Simplification Through Variable Transformations

Equation (83) or (23) can be further simplified by making a substitution in the first term,

$$y' = \frac{x + r_j + \eta_j''}{2} \tag{85}$$

which implies

$$\begin{aligned} d\eta_j'' &= 2dy' && (86)\\ \eta_j'' &= 2y' - x - r_j && (87)\\ \frac{x + r_j - \eta_j''}{2} &= x + r_j - y' && (88) \end{aligned}$$

The new integration limits can be set by noting that when $\eta_j'' = x - r_j$, then $y' = x$, and when $\eta_j'' = -x - r_j$, then $y' = 0$. Equation (84) now becomes

$$\begin{aligned} \psi_j(x, r_j) &= \exp\Big[-\int_0^x dy' a_j(x + r_j - y') \Big] \psi_j(0, x + r_j) \\ &\quad + \frac{1}{2} \int_{-x-r_j}^{x-r_j} d\eta_j' \exp\Big[-\frac{1}{2} \int_{\eta_j'}^{x-r_j} d\eta_j'' a_j\big(\frac{x + r_j - \eta_j''}{2}\big) \Big] \\ &\quad \times q_j\big(\frac{x + r_j + \eta_j'}{2}, \frac{x + r_j - \eta_j'}{2}\big) \end{aligned} \tag{89}$$

Making another substitution, $y = \frac{x + r_j + \eta_j'}{2}$, results in

$$\begin{aligned} \psi_j(x, r_j) &= \exp\Big[-\int_0^x dy' a_j(x + r_j - y') \Big] \psi_j(0, x + r_j) \\ &\quad + \int_0^x dy \exp\Big[-\frac{1}{2} \int_{2y-x-r_j}^{x-r_j} d\eta_j'' a_j\big(\frac{x + r_j - \eta_j''}{2}\big) \Big] \\ &\quad \times q_j\big(y, x + r_j - y\big) \end{aligned} \tag{90}$$

Again substitute $y' = \frac{x + r_j + \eta_j''}{2}$, resulting in

$$\begin{aligned}\psi_j(x, r_j) &= \exp\Big[-\int_0^x dy' a_j(x + r_j - y')\Big]\psi_j(0, x + r_j) \\ &\quad + \int_0^x dy \exp\Big[-\int_y^x dy' a_j(x + r_j - y')\Big] q_j(y, x + r_j - y) \qquad (91)\end{aligned}$$

A.2 Perturbative Solution

If only transport over short distances is initially considered, equation (91) can be greatly simplified by making a series of approximations. Arbitrarily large distances can then be obtained by iterating the result. To simplify the notation, define

$$\begin{aligned}a_j(y') &\equiv a_j(x + h + r_j - y') \\ q_j(y) &\equiv q_j(y, x + h + r_j - y) \qquad (92)\end{aligned}$$

Now let $x \to x + h$ in equation (91), where h is a small distance. See [2, 14, 15] for a more detailed discussion of what is meant by small. We have

$$\begin{aligned}\psi_j(x + h, r_j) &= \exp\Big[-\int_0^{x+h} dy' a_j(y')\Big]\psi_j(0, x + h + r_j) \\ &\quad + \int_0^{x+h} dy \exp\Big[-\int_y^{x+h} dy' a_j(y')\Big] q_j(y) \qquad (93)\end{aligned}$$

This equation can now be simplified in the following manner.

$$\begin{aligned}
\psi_j(x+h,r_j) &= \exp\Big[-\Big(\int_0^x dy' + \int_x^{x+h} dy'\Big)a_j(y')\Big]\psi_j(0,x+h+r_j) \\
&\quad +\Big(\int_0^x dy + \int_x^{x+h} dy\Big)\exp\Big[-\Big(\int_y^x dy' + \int_x^{x+h} dy'\Big)a_j(y')\Big]q_j(y) \\
&= \exp\Big[-\int_x^{x+h} dy' a_j(y')\Big]\exp\Big[-\int_0^x dy' a_j(y')\Big]\psi_j(0,x+h+r_j) \\
&\quad +\Big(\int_x^{x+h} dy + \int_0^x dy\Big)\exp\Big[-\int_x^{x+h} dy' a_j(y')\Big] \\
&\quad \times \exp\Big[-\int_y^x dy' a_j(y')\Big]q_j(y) \\
&= \exp\Big[-\int_x^{x+h} dy' a_j(y')\Big]\Bigg\{\mathbf{exp}\Big[-\int_{\mathbf{0}}^{\boldsymbol{x}} \boldsymbol{dy' a_j(y')}\Big]\boldsymbol{\psi_j(0,x+r_j+h)} \\
&\quad +\Big(\int_x^{x+h} dy + \int_{\mathbf{0}}^{\boldsymbol{x}} \boldsymbol{dy}\Big)\exp\Big[-\int_y^x dy' a_j(y')\Big]q_j(y)\Bigg\} \qquad (94)
\end{aligned}$$

Recognizing that equation (91) implies

$$\begin{aligned}
\psi_j(x,r_j+h) &= \exp\Big[-\int_0^x dy' a_j(y')\Big]\psi_j(0,x+r_j+h) \\
&\quad +\int_0^x dy \exp\Big[-\int_y^x dy' a_j(y')\Big]q_j(y) \qquad (95)
\end{aligned}$$

allows the two boldface pieces to be added, resulting in the following equation:

$$\begin{aligned}
\psi_j(x+h,r_j) &= \exp\Big[-\int_x^{x+h} dy' a_j(y')\Big]\Bigg\{\psi_j(x,r_j+h) \\
&\quad +\int_x^{x+h} dy \exp[-\int_y^x dy' a_j(y')]q_j(y)\Bigg\} \\
&= \exp\Big[-\int_x^{x+h} dy' a_j(y')\Big]\psi_j(x,r_j+h) \\
&\quad +\int_x^{x+h} dy \exp\Big[-\int_y^{x+h} dy' a_j(y')\Big]q_j(y) \qquad (96)
\end{aligned}$$

Expanding this equation by using the definitions given in equation (92) yields

$$\psi_j(x+h,r_j) = \exp\Big[-\int_x^{x+h} dy' a_j(x+h+r_j-y')\Big]\psi_j(x,r_j+h)$$
$$+\int_x^{x+h} dy \exp\Big[-\int_y^{x+h} dy' a_j(x+h+r_j-y')\Big] q_j(y,x+h+r_j-y) \tag{97}$$

Now in the first term, make the substitution $t' = y' - x$, and in the second term, make the substitution $t = y - x$. Then,

$$\psi_j(x+h,r_j) = \exp\Big[-\int_0^{h} dt' a_j(r_j+h-t')\Big]\psi_j(x,r_j+h)$$
$$+\int_0^{h} dt \exp\Big[-\int_{t+x}^{x+h} dy' a_j(x+h+r_j-y)\Big] q_j(t+x,r_j+h-t) \tag{98}$$

Make another substitution in the second term $t' = y' - x$.

$$\begin{aligned}\psi_j(x+h,r_j) &= \exp\Big[-\int_0^{h} dt' a_j(r_j+h-t')\Big]\psi_j(x,r_j+h) \\ &\quad +\int_0^{h} dt \exp\Big[-\int_t^{h} dt' a_j(r_j+h-t')\Big] q_j(t+x,r_j+h-t)\end{aligned} \tag{99}$$

Since h can be arbitrarily small, a series of approximations can be made to simplify equation (99) by expanding the integrand in a Taylor series and keeping terms up to first order in h. This approximation is similar to the approximations made in equations (4) and (5),

$$\int_t^{h} dt' a_j(r_j+h-t') \approx (h-t) a_j(r_j) \tag{100}$$

Since h is small, and t ranges from 0 to h in the integral, t is also small. The source term

$q_j(x+t, r_j+h-t)$ can therefore be approximated for small t.

$$\begin{aligned} q_j(x+t, r_j+h-t) &\approx q_j(x, r_j+h) + \frac{\partial q_j(x+t, r_j+h-t)}{\partial t}|_{t=0}\, t \\ &\approx q_j(x, r_j+h) + \Delta q_j t \end{aligned} \tag{101}$$

where Δq_j is an approximation of $\frac{\partial q_j(x+t, r_j+h-t)}{\partial t}|_{t=0}$, with units of $\frac{q_j}{h}$, given by the following equation:

$$\Delta q_j \equiv \frac{q_j(x, r_j+h) - q_j(x-h, r_j)}{h} \tag{102}$$

Using equations (100) and (101) in equation (99) and defining $a = a_j(r_j)$ yields

$$\begin{aligned} \psi_j(x+h, r_j) &= \exp(-ah)\psi_j(x, r_j+h) \\ &\quad + \int_0^h dt \exp[a(t-h)][q_j(x, r_j+h) + \Delta q_j t] \\ &= \exp(-ah) \times \{\psi_j(x, r_j+h) \\ &\quad + \int_0^h dt \exp(at)[q_j(x, r_j+h) + \Delta q_j t]\} \end{aligned}$$

Finally,

$$\begin{aligned} \psi_j(x+h, r_j) &= \exp(-ah)\Bigg\{\psi_j(x, r_j+h) + q_j(x, r_j+h)\frac{\exp(ah)-1}{a} \\ &\quad + \Delta q_j\left[\exp(ah)\left(\frac{h}{a} - \frac{1}{a^2}\right) + \frac{1}{a^2}\right]\Bigg\} \end{aligned} \tag{103}$$

which is equation (24).

B Cross-Section Definitions

There is no standard notation for cross sections. Below are some commonly used definitions of cross sections. The last ones are the convention used throughout this work.

The total cross section is defined as [22, 23]

$$\sigma_{\text{total}} \equiv \sigma_{\text{elastic}} + \sigma_{\text{inelastic}} + \sigma_{\text{absorption}} \tag{104}$$

The scattering cross section is defined as [2, 23, 24]

$$\sigma_{\text{scattering}} \equiv \sigma_{\text{elastic}} + \sigma_{\text{inelastic}} \tag{105}$$

giving

$$\sigma_{\text{total}} \equiv \sigma_{\text{scattering}} + \sigma_{\text{absoption}} \tag{106}$$

The reaction cross section is defined as [23]

$$\sigma_{\text{reaction}} \equiv \sigma_{\text{inelastic}} + \sigma_{\text{absoption}} \tag{107}$$

giving

$$\sigma_{\text{total}} \equiv \sigma_{\text{elastic}} + \sigma_{\text{reaction}} \tag{108}$$

Fernow [25] writes

$$\sigma_{\text{total}} \equiv \sigma_{\text{elastic}} + \sigma_{\text{quasielastic}} + \sigma_{\text{absoption}} \tag{109}$$

Rolnick [26] writes

$$\sigma_{\text{total}} \equiv \sigma_{\text{elastic}} + \sigma_{\text{inelastic}} \tag{110}$$

where $\sigma_{\text{inelastic}} = \sigma_{\text{reaction}}$, in other words, where all interactions except elastic scattering are included in $\sigma_{\text{inelastic}}$. The convention of Rolnick is used throughout this paper.

C Two-Body Decays

Recall that all energies are kinetic energies and that $c = 1$. The reaction to be considered here is of the type $\pi \to \mu + \nu$. A general two-body decay can be treated simply by the appropriate replacements of π, μ, and ν. The production of muons will be treated here. Neutrinos will be considered only as they pertain to muon production.

In a two-body decay, conservation of energy and momentum is enough to completely determine the kinematics. In the rest frame of the original particle a, production of particle b is isotropic, and its energy is [11]

$$\begin{aligned} E_\mu^{rf} &= \frac{m_\pi^2 + m_\mu^2 - m_\nu^2}{2m_\pi} - m_\mu \\ &\equiv E^* \end{aligned} \tag{111}$$

In any other reference frame, energy is a function of angle that can be determined by a simple Lorentz transformation.

$$E' = \gamma(E + m + \beta p \cos\theta) - m \tag{112}$$

The rate of decay is independent of the medium, so the production cross section is dependent only on the difference of the two angles $\widehat{\Omega} \equiv \widehat{\Omega}_\pi - \widehat{\Omega}_\mu$. Consider the following

equation:

$$u_{\mu\pi}(\widehat{\Omega}_\mu, E_\mu, E_\pi)\phi_\pi(\overrightarrow{x}, E_\pi)dE_\mu\, d^2\widehat{\Omega}_\mu\, d^3\overrightarrow{x} = d^6N_{\mu\pi}(E_\pi) \tag{113}$$

where $u_{\mu\pi}(\widehat{\Omega}_\mu, E_\mu, E_\pi)$, which has the dimensions of $\frac{1}{\text{length}\cdot\text{energy}\cdot\text{solid angle}}$, is the macroscopic cross section for the production of muons from the decay of pions; $d^6N_{\mu\pi}$, which has units $\frac{1}{\text{energy}}$, is the number of muons with energies between E_μ and $E_\mu + dE_\mu$ moving between directions $\widehat{\Omega}_\mu$ and $\widehat{\Omega}_\mu + d\widehat{\Omega}_\mu$ produced from pions of energy E_π in the region between $\overrightarrow{x}$ and $\overrightarrow{x} + d\overrightarrow{x}$. $\phi_\pi(\overrightarrow{x}, E_\pi)$, which has the dimensions of $\frac{1}{\text{area}\,\text{energy}}$, is the flux of pions of energy E_π at position $\overrightarrow{x}$. The cross section is independent of the direction of the pion, so only fluxes independent of pion direction need be considered. Now, also consider that

$$N_{\mu\pi}(E_\pi) = \frac{\Gamma_{\mu\pi}}{\Gamma}\int d^3\overrightarrow{x}\,\frac{\phi_\pi(\overrightarrow{x}, E_\pi)}{\beta_\pi\gamma_\pi\tau_\pi} \tag{114}$$

where $N_{\mu\pi}(E_\pi) \equiv \int d^6N_{\mu\pi}(E_\pi)$ and $\frac{\Gamma_{\mu\pi}}{\Gamma}$ is a branching ratio; τ_π is the mean lifetime of a pion, implying that $\beta_\pi c\gamma_\pi\tau_\pi$ is the mean decay length of a pion. Together equations (113) and (114) imply that

$$\frac{\frac{\Gamma_{\mu\pi}}{\Gamma}}{\beta_\pi c\gamma_\pi\tau_\pi} = \int d^2\widehat{\Omega}_\mu dE_\mu u_{\mu\pi}(\widehat{\Omega}_\mu, E_\mu, E_\pi) \tag{115}$$

$\frac{\Gamma_{\mu\pi}}{\Gamma} = 0.999877$ [11], which will be set to unity for the rest of this appendix.

The production cross section of the muon in the rest frame of the pion can now be deduced from the previous equation.

$$u^{rf}_{\mu\pi}(\widehat{\Omega}, E_\mu, E_\pi) = \frac{\delta(E^{rf}_\mu - E^*)}{4\pi\beta^{rf}_\pi\gamma^{rf}_\pi\tau^{rf}_\pi} \tag{116}$$

where rf signifies the rest frame of the pion. This quantity can be seen to have the correct angular dependence (i.e., none), and the correct energy dependence, and it integrates to the correct result. The cross section is infinite since $\beta_\pi^{rf} = 0$ by definition. The transport equation needs cross sections in the rest frame of a medium, so a Lorentz transformation needs to be performed.

C.1 Lorentz Transformation

Consider how the following quantity transforms under Lorentz transformations:

$$u_\pi(E_\pi) = \frac{1}{\beta_\pi \gamma_\pi \tau_\pi} \tag{117}$$

Since equation (117) holds true in any reference frame, it can easily be deduced how $u_\pi(E_\pi)$ transforms under Lorentz boosts.

$$u'_\pi(E'_\pi) = \frac{\beta_\pi \gamma_\pi}{\beta'_\pi \gamma'_\pi} u_\pi(E_\pi) \tag{118}$$

Equation (115) then implies

$$\begin{aligned} \int u'_{\mu\pi}(\widehat{\Omega}', E'_\mu, E'_\pi) dE'_\mu \, d\widehat{\Omega}' &= \frac{\beta_\pi \gamma_\pi}{\beta'_\pi \gamma'_\pi} \int u_{\mu\pi}(\widehat{\Omega}, E_\mu, E_\pi) dE_\mu \, d\widehat{\Omega} \\ &= \frac{\beta_\pi \gamma_\pi}{\beta'_\pi \gamma'_\pi} \int u_{\mu\pi}[\widehat{\Omega}(\widehat{\Omega}', E'_\mu), E_\mu(\widehat{\Omega}', E'_\mu), E_\pi] \\ &\quad \times \frac{\partial(\widehat{\Omega}, E_\mu)}{\partial(\widehat{\Omega}', E'_\mu)} dE'_\mu \, d\widehat{\Omega}' \end{aligned} \tag{119}$$

where $\frac{\partial(\widehat{\Omega}, E_\mu)}{\partial(\widehat{\Omega}', E'_\mu)} = \frac{p'_\mu}{p_\mu}$ is the Jacobian of the transformation between $(\widehat{\Omega}', E'_\mu)$ and $(\widehat{\Omega}, E_\mu)$, as shown in Hagedorn [27]. Hagedorn calculates the Jacobian for the total energy not kinetic, but $dE_{\text{total}} = dE_{\text{kinetic}}$ so the Jacobian is the same.

The macroscopic cross section in a boosted frame is

$$u'_{\mu\pi}(\widehat{\Omega}', E'_\mu, E'_\pi) = u_{\mu\pi}[\widehat{\Omega}(\widehat{\Omega}', E'_\mu), E_\mu(\widehat{\Omega}', E'_\mu); E_\pi]\frac{p'_\mu}{p_\mu}\frac{\beta_\pi\gamma_\pi}{\beta'_\pi\gamma'_\pi} \tag{120}$$

The cross section in the lab frame is therefore

$$\begin{aligned} u'_{\mu\pi}(\widehat{\Omega}', E', E'_\pi) &= \frac{\delta(E - E^*)}{4\pi\beta_\pi\gamma_\pi\tau_\pi}\frac{p'}{p} \\ &= \frac{\delta[\gamma_\pi(E' + m + \beta_\pi p'\cos\theta') - E^* - m]p'}{4\pi\beta_\pi\gamma_\pi\tau_\pi\sqrt{\gamma^2(E' + m + \beta_\pi p'\cos\theta')^2 - m^2}} \end{aligned} \tag{121}$$

where all variables without subscripts from here on refer to the muon.

In the straight-ahead approximation, a macroscopic spectral distribution is needed. Simply integrate over angle

$$\begin{aligned} u_{\mu\pi}(E_\mu, E_\pi) &= \int d\widehat{\Omega} u_{\mu\pi}(\widehat{\Omega}, E_\mu, E_\pi) \\ &= 2\pi\int d(\cos\,\theta) u_{\mu\pi}(\widehat{\Omega}, E_\mu, E_\pi) \end{aligned} \tag{122}$$

Now make the substitution

$$\delta[\gamma_\pi(E + m + \beta_\pi p\cos\theta) - E^* - m] = \frac{1}{\gamma_\pi\beta_\pi p}\delta\big(\cos\theta + \frac{E + m}{\beta_\pi p} - \frac{E^* + m}{\gamma_\pi\beta_\pi p}\big) \tag{123}$$

resulting in

$$u_{\mu\pi}(\widehat{\Omega}, E, E_\pi) = \frac{\frac{1}{\gamma_\pi\beta_\pi}\delta\big(\cos\theta + \frac{E + m}{\beta_\pi p} - \frac{E^* + m}{\gamma_\pi\beta_\pi p}\big)}{4\pi\beta_\pi\gamma_\pi\tau_\pi\sqrt{\gamma_\pi^2(E + m + \beta_\pi p\cos\theta)^2 - m^2}} \tag{124}$$

where the primes have been dropped for convenience.

$$\begin{aligned} u_{\mu\pi}(E_\mu, E_\pi) &= 2\pi\int d(\cos\theta)\frac{\frac{1}{\gamma_\pi\beta_\pi}\delta\big(\cos\theta + \frac{E + m}{\beta_\pi p} - \frac{E^* + m}{\gamma_\pi\beta_\pi p}\big)}{4\pi\beta_\pi\gamma_\pi\tau_\pi\sqrt{\gamma_\pi^2(E + m + \beta_\pi p\cos\theta)^2 - m^2}} \\ &= \frac{1}{\beta_\pi\gamma_\pi\tau_\pi}\frac{1}{2\beta_\pi\gamma_\pi\sqrt{(E^* + m)^2 - m^2}} \\ &= \frac{1}{\beta_\pi\gamma_\pi\tau_\pi}\frac{1}{2\beta_\pi\gamma_\pi p^*} \end{aligned} \tag{125}$$

where E^* is defined in equation (111) and $p^* = \sqrt{(E^*+m)^2 - m^2}$. Note that the cross section $u_{\mu\pi}(E_\mu, E_\pi)$ has the above form only in the energy region between $\gamma(E^* + m - \beta p^*) - m$ and $\gamma(E^* + m + \beta p^*) - m$. It is zero outside this region due to conservation of energy.

The total cross section, which is equal to the inverse decay length for the case of a single decay channel, should be the following:

$$u_{\mu\pi}(E_\pi) = \frac{1}{\beta_\pi \gamma_\pi \tau_\pi} \tag{126}$$

therefore check that

$$\int_{E_{\min}}^{E_{\max}} dE_\mu u_{\mu\pi}(E_\mu, E_\pi) = \frac{1}{\beta_\pi \gamma_\pi \tau_\pi} \tag{127}$$

where $E_{\min}$ and $E_{\max}$ are $\gamma(E^*+m-\beta p^*)-m$ and $\gamma(E^*+m+\beta p^*)-m$, respectively. These limits are calculated by using the formula $E = \gamma(E^* + m - \beta p^* \cos\theta^*)$. The minimum energy occurs when $\theta = 180^\circ$ and the maximum occurs when $\theta = 0^\circ$.

$$\begin{aligned} \int_{\gamma(E^*+m-\beta_\pi p^*)-m}^{\gamma(E^*+m+\beta_\pi p^*)-m} dE_\mu \frac{1}{\beta_\pi \gamma_\pi \tau_\pi} \frac{1}{2\beta_\pi \gamma_\pi p^*} &= \frac{1}{\beta_\pi \gamma_\pi \tau_\pi} \frac{2\gamma_\pi \beta_\pi p^*}{2\beta_\pi \gamma_\pi p^*} \\ &= \frac{1}{\beta_\pi \gamma_\pi \tau_\pi} \end{aligned} \tag{128}$$

D MESTRN Subprogram Descriptions

Unless otherwise stated:

"Length" has the units $\frac{\text{gm}}{\text{cm}^2}$.

All energies are kinetic and in units of $\frac{\text{MeV}}{\text{amu}}$.

All fluxes are actually flux multiplied by stopping power as shown in equation (21) and have the units $\frac{1}{\text{gm}}$.

Stopping powers have the units $\frac{\text{MeV}\cdot\text{cm}^2}{\text{amu}\cdot\text{gm}}$.

Inverse mean free paths have the units $\frac{\text{cm}^2}{\text{gm}}$.

Inverse decay lengths have the units $\frac{\text{cm}^2}{\text{gm}}$.

Microscopic total cross sections have the units cm^2.

Macroscopic total cross sections have the units $\frac{\text{cm}^2}{\text{gm}}$.

Microscopic spectral distributions have the units $\frac{\text{cm}^2\cdot\text{amu}}{\text{MeV}}$.

Macroscopic spectral distributions have the units $\frac{\text{cm}^2\cdot\text{amu}}{\text{MeV}\cdot\text{gm}}$.

All target properties needed for pion transport are included in the following common statement (see HZETRN user instructions and the main program of HZETRN):

COMMON/TARGET/NLAY,XLAY,DN,NAT,ATRG,ZTRG,DENSTRG

where

NLAY is the layer number.

XLAY is the thickness of a given layer.

DN is the density of the target in $\frac{\text{gm}}{\text{cm}^3}$.

NAT is the number of elements in the target.

ATRG are the atomic numbers of the elements in the target.

ZTRG are the numbers of protons in the elements in the target.

DENSTRG is the density of the target in $\frac{\text{number}}{\text{gm}}$.

Function AAMMCS(EPROJ,EPI,P,J): This function calculates the macroscopic spectral distribution $\sigma_{\pi A}(E_\pi, E_A)$ for production of pions of type J with energy EPI from ions of type P with energy EPROJ interacting with the medium. It has the following real input:

EPROJ is the energy of the projectile nucleus.

EPI is the energy of the pion being produced,

and the following integer input:

P labels the type of nucleus projectile.

J labels the type of pion produced.

The function AAMMCS calls the following functions:

AAPIP returns the microscopic spectral distribution for positive pion production in nucleus-nucleus collisions.

AAPIM is the same as function AAPIP except that it is for negative pion production.

A returns the atomic number of ions. It is an entry in the HZETRN function TS.

Function AAPIM(EPROJ,EPI,APROJ,AT): This function returns the microscopic spectral distribution $\frac{d\sigma}{dE}$ for the reaction $A + A \rightarrow \pi^- + x$.

It has the following real input:

EPROJ is the energy of the projectile.

EPI is the energy of the produced pion.

APROJ is the atomic number of projectile.

AT is the atomic number of target.

The function AAPIM calls the following functions:

PIMINUS function returns the microscopic spectral distribution for reaction $proton + proton \rightarrow \pi^- + x$

Function AAPIP(EPROJ,EPI,APROJ,AT): This function returns the microscopic spectral distribution $\frac{d\sigma}{dE}$ for the reaction $A + A \rightarrow \pi^+ + x$. It has the same real input as the function AAPIM.

The function AAPIP calls the following function:

PIPLUS returns the microscopic spectral distribution for the reaction $proton + proton \rightarrow \pi^+ + x$.

Function ANUMM(J): This function returns the dimensionless factor that scales the stopping power of pions and muons of type J from that of protons.

It has the following integer input:

J is the index for pion/muon type (J = 1,2,3 or 4 correspond to μ^+, μ^-, π^+, and π^-, respectively).

Function INELASTICPIMA(ATARG,EPI): This function returns microscopic total (elastic + inelastic) pion-nucleus interaction cross sections $\sigma_\pi(E)$ for the reaction $\pi^- + A \rightarrow x$. It is temporarily set to return the inelastic cross section as discussed in section 4.

Real input:

AT is the atomic number of the target nucleus.

EPI is the energy of the pion.

Function INELASTICPIPA(ATARG, EPI): This function returns microscopic total (elastic + inelastic) pion-nucleus interaction cross sections $\sigma_\pi(E)$ for the reaction $\pi^+ + A \rightarrow x$. It is temporarily set to return the inelastic cross section as discussed in section 4.

Real input:

AT is the atomic number of the target nucleus.

EPI is the energy of the pion.

Function MAMMCS(EPROJ,EPI,P,J): This function returns macroscopic spectral distribution $\sigma_{\pi\pi}(E_\pi, E_\pi)$ for production of pions from collisions of pions with the medium.

It has the following real input:

EPROJ is the energy of the projectile pion.

EPI is energy of the produced pion,

and the following integer input:

P labels the type of pion projectile.

J labels the type of pion produced.

The function MAMMCS calls the following functions:

PIPAPIP returns microscopic spectral distribution for the reaction $\pi^+ + A \to \pi^+ + x$.

PIMAPIP is the same as PIPAPIP except it is for the reaction $\pi^- + A \to \pi^+ + x$.

PIPAPIM is the same as PIPAPIP except it is for the reaction $\pi^+ + A \to \pi^- + x$.

PIMAPIM is the same as PIPAPIP except it is for the reaction $\pi^- + A \to \pi^- + x$.

Function MGAUSSMM(A,B,N,SOURCE,J,EPI,E,PSI,PSIMM0,II,IJ,IM): This function is the Gaussian integration function mgauss slightly modified to integrate the function source over projectile energy.

It has the following real input:

A is the lower integration limit.

B is the upper limit.

SOURCE is the function to be integrated.

E is the energy array.

EPI is the energy of pion/muon produced.

PSI is the ion flux.

PSIMM0 is the pion/muon flux.

The following integer input is used:

J labels pion/muon type being produced.

II is the number of points in the energy array.

IJ is the number of ion types.

IM is the number of pion/muon types.

N is number of subintervals for the integration.

The function MGAUSSMM calls the following function:

SOURCE is the function to be integrated.

Function MMLOSS(EPI,J): This function returns the inverse mean free path for pion and muons of type J and of energy EPI.

Real input:

EPI is the energy.

Integer Input:

J is the particle type: 1 is μ^+; 2 is μ^-; 3 is π^+; and 4 is π^-.

The function MMLOSS calls the following functions:

PIDECAY calculates the pion/muon inverse decay length.

PNST returns the microscopic total (elastic + inelastic) pion-nucleus interaction cross section.

Subroutine MULIMITS(EMU,THRESHOLD,UPPERLIM): This subroutine calculates integration limits for muon production from pion decay.

Real input:

EMU is the energy of the muon created.

THRESHOLD is the lower limit.

UPPERLIM is the upper limit.

Function MUPRODPI(EPROJ,EMU): This function returns the macroscopic spectral distribution $u_{\mu\pi}(E_\mu, E_\pi)$ for production of muons from the decay of like charged pions, that is, for the decay $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$.

Real input:

EPROJ is the energy of the projectile pion that is decaying.

EMU is the energy of the muon created.

The function MUPRODPI calls the following function:

PIDECAY returns the inverse decay length of the pion.

Subroutine OUTPUT: This function is temporarily used to output fluxes. It will be removed when pion/muon transport is fully integrated into HZETRN.

Subroutine PGTMM (II,IJ,E,R,ST,PSI,PSIMM,H,QMOLD): This subprogram propagates the pion and muon components of radiation a distance h.

It has the following real input:

E is the energy array.

R is the proton range array.

ST is the proton stopping power array.

PSI are ion fluxes.

PSIMM are pion/muon fluxes.

H is the distance transported for one call of the subroutine.

QMOLD holds array qm that was calculated during the last iteration

and uses the following integer input:

II is the number of energy/range grid points.

IJ is the number of ions being transported.

The function PGTMM calls the following functions:

ANUMM calculates stopping power scaling factors for mesons.

MMLOSS calculates inverse of the mean free path of pions and muons.

PHI interpolates flux arrays. It is in the original HZETRN.

SOURCE calculates pion and muon sources at specific energies.

MGAUSSMM integrates the function source over projectile energy.

Function PIDECAY(EPI,J): This function returns inverse decay length $u_j(E)$ of pions or muons.

Real input:

EPI is energy.

Integer input:

J is the particle type: 1 is μ^+; 2 is μ^-; 3 is π^+; and 4 is π^-.

Function PIMAPIM(EPROJ,EPI,AT): This function returns microscopic spectral distribution $\frac{d\sigma}{dE}$ for the reaction $\pi^- + A \to \pi^- + x$.

Real input:

EPROJ is the energy of projectile pion.

EPI is the energy of produced pion.

AT is the atomic number of target.

Function PIMAPIP(EPROJ,EPI,AT): Same as PIMAPIM but for the reaction $\pi^- + A \to \pi^+ + x$.

Function PIMINUS(EPR,E): This function returns microscopic spectral distribution $\frac{d\sigma}{dE}$ for negative pion production from proton-proton collisions, that is, for the reaction $p + p \to \pi^- + x$.

Real Input:

EPR is the energy of the projectile proton.

E is the energy of pion.

Function PIPAPIM(EPROJ,EPI,AT): Same as PIMAPIM but for the reaction $\pi^+ + A \rightarrow \pi^- + x$.

Function PIPAPIP(EPROJ,EPI,AT): Same as PIMAPIM but for the reaction $\pi^+ + A \rightarrow \pi^+ + x$.

Function PIPLUS(EPR,EPI): This function returns the microscopic spectral distribution $\frac{d\sigma}{dE}$ for positive pion production from proton-proton collisions, that is, for the reaction $p + p \rightarrow \pi^+ + x$.

Real input:

EPR is the energy of the projectile proton.

EPI is the energy of the pion.

Function SOURCE(EPI,EPROJ,E,PSI0,PSIMM0,J,II,IJ,IM): This function calculates pion and muon sources at specific energies.

Real input:

E is the energy grid.

PSI0 is the ion flux array.

PSIMM0 is the pion/muon flux array.

EPI is the energy of the pion/muon produced.

EPROJ is the energy of the projectile.

Integer input:

II is the number of range grid points.

IJ is the total number of ions.

IM is the total number of pion/muon types.

J labels the type of pion/muon being produced.

The function SOURCE calls the following functions:

AAMMCS returns the macroscopic cross sections for production of pions from collisions of nuclei with the medium.

MAMMCS returns the macroscopic cross section for production of pions from collisions of pions with the medium.

MUPRODPI returns the macroscopic cross section for the production of muons from the decay of like charged pions.

PHI interpolates fluxes. It is in the original HZETRN.

D.1 Summary of Functions and Subroutines

1. AAMMCS: macroscopic spectral distribution $\sigma_{\pi A}(E_\pi, E_A)$.

2. AAPIM: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $A + A \rightarrow \pi^- + x$.

3. AAPIP: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $A + A \rightarrow \pi^+ + x$.

4. ANUMM: factor that scales pion and muon stopping powers from proton stopping powers.

5. INELASTICPIMA: microscopic total cross section $\sigma_\pi(E)$ for $\pi^- + A \rightarrow x$.

6. INELASTICPIPA: microscopic total cross section $\sigma_\pi(E)$ for $\pi^+ + A \rightarrow x$.

7. MAMMCS: macroscopic spectral distribution $\sigma_{\pi\pi}(E_\pi, E_\pi)$.

8. MGAUSSMM: integrates function SOURCE over energy.

9. MMLOSS: inverse mean free path $a_j(E)$ of pions and muons.

10. MUPRODPI: macroscopic spectral distribution $u_{\mu\pi}$ for $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$.

11. OUTPUT: temporary subroutine used to output fluxes.

12. PGTMM: main propagation subroutine for pions/muons.

13. PIDECAY: inverse mean decay length for pion/muon $u_j(E)$.

14. PIMAPIM: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $\pi^- + A \rightarrow \pi^- + x$.

15. PIMAPIP: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $\pi^- + A \to \pi^+ + x$.

16. PIMINUS: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $p + p \to \pi^- + x$.

17. PIPAPIM: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $\pi^+ + A \to \pi^- + x$.

18. PIPAPIP: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $\pi^+ + A \to \pi^+ + x$.

19. PIPLUS: microscopic spectral distribution $\frac{d\sigma}{dE}$ for $p + p \to \pi^+ + x$.

20. SOURCE: calculates pion and muon sources.

E The Meson and Muon Transport Code MESTRN

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The complete Fortran subprograms that make up the transport code MESTRN are contained in this appendix. In this code all energies are kinetic. Also, cross-section functions were named after the reactions. For example, AAMMCS returns cross sections for the reactions involving $AA \rightarrow$ Meson/Muon + x.

```
      REAL FUNCTION AAMMCS(EPROJ, EPI, P, J)
**********************************************************************
* This function calculates the the macroscopic spectral distribution
* for production of pions/muons of type J from ions of type P.
**********************************************************************
*
      IMPLICIT NONE
*
**********************************************************************
```

```
* Integer Variables
*  NAT  number of different types of atoms in medium
*  P    labels type of ion projectile
*  J    labels type of meson being produced
*  K    loop index
************************************************************************
      INTEGER NAT, P, K, J
************************************************************************
* Real Variables
*  DENSTRG  density of atoms of type K in medium
*  ATRG     atomic number of those atoms
*  EPROJ    energy of projectile
*  EPI      energy of meson being produced
*  AAPIP    function that returns microscopic spectral distribution
*           for positive pion production in nucleus-nucleus
*           collisions
*  AAPIM    same as AAPIP but for negative pion production
*  A        function that returns atomic number it is an entry in
*           function TS
************************************************************************
      REAL DENSTRG(5),EPROJ,EPI,SIGMA,A,AAPIP,AAPIM,ATRG(5)
```

```
*********************************************************************
* unused reals for common statement
*********************************************************************
      REAL NLAY,XLAY,DN,ZTRG(5)
*********************************************************************
* set commons
*********************************************************************
      COMMON/TARGET/NLAY,XLAY,DN,NAT,ATRG,ZTRG,DENSTRG
*********************************************************************
*--------------------------------------------------------------------
*-------------------------start function aammcs----------------------
*--------------------------------------------------------------------
*********************************************************************
*     initialize aammcs
*********************************************************************
*
      AAMMCS=0.0
*
*********************************************************************
* First consider positive pions
*********************************************************************
```

```
*
      IF (J .EQ. 3) THEN
*
**********************************************************************
* sum micro-cross section over target atoms to get macroscopic cross
* section
**********************************************************************
*
         DO 10 K=1,NAT
*
**********************************************************************
* calculation based on scaling from pp collisions to AA collisions
**********************************************************************
*
            AAMMCS= AAMMCS + DENSTRG(K)*
     $           AAPIP(EPROJ,EPI,A(P),ATRG(K))
*
**********************************************************************
* end sum over target atoms
**********************************************************************
*
```

```
 10      CONTINUE
*
**********************************************************************
* For negative pions
**********************************************************************
*
      ELSE IF (J .EQ. 4) THEN
*
**********************************************************************
* sum micro over target atoms to get macroscopic cross section
**********************************************************************
*
         DO 20 K=1,NAT
*
**********************************************************************
* calculation based on scaling from pp collisions to AA collisions
**********************************************************************
*
            AAMMCS= AAMMCS+DENSTRG(K)*
     $           AAPIM(EPROJ,EPI,A(P),ATRG(K))
*
```

```
************************************************************************
* end sum for negative pions
************************************************************************
*
 20      CONTINUE
*
************************************************************************
* error statement for debugging
************************************************************************
*
      ELSE
         PRINT*, "error in aammcs there should be no else"
*
************************************************************************
* end function AAMMCS
************************************************************************
      END IF
      RETURN
      END
************************************************************************
************************************************************************
```

```
      REAL FUNCTION AAPIM(EPROJ,EPI,APROJ,AT)
*********************************************************************
* this function that returns microscopic spectral distribution for
* the reaction A + A -> pi- + x.
*********************************************************************
*
      IMPLICIT NONE
*
*********************************************************************
      INTEGER A
*********************************************************************
* Real Variables
*  EPROJ   kinetic energy of projectile nucleus
*  EPI     kinetic energy of produced   pi+
*  PIMINUS function returns spectral distribution for reaction
*          proton + proton -> pi- +x
*  ALPHA   scales collisions from target nucleon to nucleus
*  APROJ   atomic number of projectile
*  AT      atomic number of target
*********************************************************************
      REAL EPROJ, EPI, PIMINUS, ALPHA, APROJ, AT
```

```
************************************************************************
*-----------------------------------------------------------------------
*-------------------------start function AAPIM-------------------------
*-----------------------------------------------------------------------
************************************************************************
* set alpha.
************************************************************************
*
      ALPHA = 0.69
*
************************************************************************
* calculate cross section
************************************************************************
*
      AAPIM=PIMINUS(EPROJ,EPI)*(AT*APROJ)**ALPHA
*
************************************************************************
* end function aapim
************************************************************************
      RETURN
      END
```

```
************************************************************************
************************************************************************

      REAL FUNCTION AAPIP(EPROJ,EPI,APROJ,AT)
************************************************************************
* this function returns microscopic spectral distribution for
* the reaction A + A -> pi+ + x.
************************************************************************
*
      IMPLICIT NONE
*
************************************************************************
* Real Variables
*  EPROJ   kinetic energy of projectile nucleus
*  EPI     kinetic energy of produced   pi+
*  PIPLUS function returns spectral distribution for reaction
*          proton + proton -> pi+ +x
*  ALPHA   scales collisions from target nucleon to nucleus
*  APROJ   atomic number of projectile
*  AT      atomic number of target
************************************************************************
```

```
      REAL EPROJ, EPI, PIPLUS, ALPHA, AT, APROJ
*********************************************************************
*-------------------------------------------------------------------
*-------------------------start function AAPIP-----------------------
*-------------------------------------------------------------------
*********************************************************************
* set alpha.
*********************************************************************
*
      ALPHA = 0.69
*
*********************************************************************
* calculate cross section
*********************************************************************
*
      AAPIP=PIPLUS(EPROJ,EPI)*(AT*APROJ)**ALPHA
*
*********************************************************************
* end function aapip
*********************************************************************
      RETURN
```

```
      END
*********************************************************************
*********************************************************************
      REAL FUNCTION ANUMM(J)
*********************************************************************
* This function inputs the index for meson type (J = 1,2,3 or 4,
* mu+, mu-, pi+, and pi- respectively) and returns the factor ANUMM
* which scales the stopping power of mesons and muons to that of
* protons.
*********************************************************************
*
      IMPLICIT NONE
*
*********************************************************************
* Integer variables
*  J   labels meson type
*  IM  number of meson types
*  ZPI labels meson charge
*********************************************************************
      INTEGER ZPI, J, IM
      PARAMETER (IM=4)
```

```
      DIMENSION ZPI(IM)
      DATA ZPI/1, -1,  1, -1/
*********************************************************************
* Real Variables
* API is mass in amu
*********************************************************************
      REAL API(IM)
      DATA API /0.11342, 0.11342, 0.14983, 0.14983/
*********************************************************************
* calculate
*********************************************************************
*
      ANUMM=(ZPI(J))**2/API(J)
*
*********************************************************************
* end function anumm
*********************************************************************
      RETURN
      END
*********************************************************************
*********************************************************************
```

```
      REAL FUNCTION INELASTICPIMA(ATARG, EPI)
*********************************************************************
      IMPLICIT NONE
*********************************************************************
      REAL EPI, EPIGEV, PLAB, MPIGEV, API, X, A(11), ATARG,
     $     GAUSSA,GAUSSB,LORA,LORB,S,B(6),MP
*********************************************************************
      DATA A /31.9,  -1.27, 0.176,  23.5, 1.32, 2.37,
     $     12.5, 0.0745,  7.73, -0.33,  0.0518/
      DATA B /10.9, 20.2, 3.90,  0.093, 0.358, 0.560/
      MPIGEV=.139570
      API=0.14983
      MP=0.939272
*********************************************************************
* change units of energy to mev and calculate momentum
*********************************************************************
      EPIGEV=EPI*API/1000.0
      PLAB=SQRT(EPIGEV**2.0-EPIGEV*MPIGEV+2.0*MPIGEV**2.0)
*********************************************************************
* calculate cross section
*********************************************************************
```

```
      X=LOG(PLAB);
      IF (plab < 5.02234) THEN
         gaussa= A(1) * exp(-0.5*((x-A(2))/A(3))**2.0)
         gaussb= A(4) * exp(-0.5*((x-A(5))/A(6))**2.0)
         lora=A(7)/(1.0+(x/A(8))**2.0)
         lorb=A(9)/(1.0+((x-A(10))/A(11))**2.0)
         INELASTICPIMA=gaussa+ gaussb + lora +lorb
      ELSE
         s=mpiGEV*mpiGEV + mp*mp +2.0*sqrt(plab*plab
     $        + mpiGEV*mpiGEV)*mp;
         INELASTICPIMA=B(1)*s**B(4) + B(2)*s**(-B(5))
     $        + B(3)*s**(-B(6))
      END IF

*********************************************************************
* change cross section units from mb to cm^2 and scale to
* pion nucleus collision
*********************************************************************

      INELASTICPIMA=INELASTICPIMA*1E-27*ATARG**0.73
```

```
************************************************************************
* end function inelasticpima
************************************************************************
      RETURN
      END
************************************************************************
************************************************************************
      REAL FUNCTION INELASTICPIPA(ATARG, EPI)
************************************************************************
      IMPLICIT NONE
************************************************************************
      REAL EPI, EPIGEV, PLAB, MPIGEV, API, X, A(6), ATARG
************************************************************************
      DATA A /20.1, -0.0649, 0.197, 1.18, 0.866, 0.712/
      MPIGEV=.139570
      API=0.14983
************************************************************************
* change units of energy to GeV and calculate momentum
************************************************************************
      EPIGEV=EPI*API/1000.0
```

```
      PLAB=SQRT(EPIGEV**2.0-EPIGEV*MPIGEV+2.0*MPIGEV**2.0)
*********************************************************************
* calculate cross section
*********************************************************************

      X=LOG(PLAB);
      INELASTICPIPA=A(1)/(1.0 + exp((A(2)-x)/A(3)))
     $     + A(4)*exp(-0.5*(((x-A(5))/A(6))**2.0))

*********************************************************************
* change cross section units from mb to cm^2 and scale to
* pion nucleus collision
*********************************************************************

      INELASTICPIPA=INELASTICPIPA*1E-27*ATARG**0.73

*********************************************************************
* end function inelasticpipa
*********************************************************************
      RETURN
      END
```

```
************************************************************************
************************************************************************
      REAL FUNCTION MAMMCS(EPROJ, EPI, P, J)
************************************************************************
* this function returns macroscopic cross section for production
* of mesons from collisions of mesons with medium. Production from
* decay is not included.
************************************************************************
*
      IMPLICIT NONE
*
************************************************************************
* Integer Variables
*  NAT  number of different types of atoms in medium

*  P    labels type of meson projectile
*  J    labels type of meson being produced
*  K    loop index
************************************************************************
      INTEGER NAT, P, K, J
************************************************************************
```

```
* Real Variables
*  ATRG atomic number of those atoms
*  DENSTRG  density of atoms of type K in medium
*  EPROJ    energy of projectile
*  EPI      energy of meson being produced
*  PIPAPIP  function that returns microscopic spectral distribution
*           for the reaction pi+ + A -> pi+ + x
*  PIMAPIP  same as PIPAPIP but for the reaction  pi- + A -> pi+ + x
*  PIPAPIM  same as PIPAPIP but for the reaction  pi+ + A -> pi- + x
*  PIMAPIM  same as PIPAPIP but for the reaction  pi- + A -> pi- + x
*****************************************************************
      REAL DENSTRG(5),EPROJ,EPI,PIPAPIP,PIMAPIP,
     $     ATRG(5),PIPAPIM,PIMAPIM
*****************************************************************
* unused reals for common statement
*****************************************************************
      REAL NLAY,XLAY,DN,ZTRG(5)
*****************************************************************
* set common variables
*****************************************************************
      COMMON/TARGET/NLAY,XLAY,DN,NAT,ATRG,ZTRG,DENSTRG
```

```
*********************************************************************
*-------------------------------------------------------------------
*---------------------------------start function--------------------
*-------------------------------------------------------------------
*********************************************************************
* initialize mammcs
*********************************************************************
*
      MAMMCS=0.0
*
*********************************************************************
* First consider production of positive pions
*********************************************************************
*
      IF (J .EQ. 3) THEN
*
*********************************************************************
* sum micro-cross section over target atoms to get macroscopic
* cross sections
*********************************************************************
*
```

```
         DO 30 K=1,NAT
*
**********************************************************************
* calculate microscopic cross sections.
* kaons are not included so no source from decay.
**********************************************************************
* for the reaction  pi+ + A -> pi+ x
**********************************************************************
*
            IF (P .EQ. 3) THEN
               MAMMCS= MAMMCS + DENSTRG(K)*
     $              PIPAPIP(EPROJ,EPI,ATRG(K))
*
**********************************************************************
* for the reaction  pi- + A -> pi+ x
**********************************************************************
*
            ELSE IF (P .EQ. 4) THEN
               MAMMCS= MAMMCS + DENSTRG(K)*
     $              PIMAPIP(EPROJ,EPI,ATRG(K))
*
```

```
************************************************************************
* end sum over target atoms
************************************************************************
*
            END IF
 30      CONTINUE
*
************************************************************************
* Now consider production of negative pions
************************************************************************
*
      ELSE IF (J .EQ. 4) THEN
*
************************************************************************
* sum micro-cross section over target atoms.
************************************************************************
*
         DO 40 K=1,NAT
*
************************************************************************
* calculate microscopic cross sections
```

```
* Also kaons are not included so no source from decay
*********************************************************************
* for the reaction  pi+ + A -> pi- x
*********************************************************************
*
            IF (P .EQ. 3) THEN
               MAMMCS= MAMMCS + DENSTRG(K)*
     $              PIPAPIM(EPROJ, EPI, ATRG(K))
*
*********************************************************************
* for the reaction  pi- + A -> pi- x
*********************************************************************
*
            ELSE IF (P .EQ. 4) THEN
               MAMMCS= MAMMCS + DENSTRG(K)*
     $              PIMAPIM(EPROJ,EPI,ATRG(K))
*
*********************************************************************
* end sum
*********************************************************************
*
```

```
            END IF
 40      CONTINUE
*
*****************************************************************
* end function mammcs
*****************************************************************
      END IF
      RETURN
      END
*****************************************************************
*****************************************************************

      REAL FUNCTION MGAUSSMM(A,B,N,SOURCE,J,EPI,E,FLUX,FLUXMM,II,
     $     IJ,IM)
*****************************************************************
* This function is the Gaussian integration function mgauss
* modified to integrate the function source over projectile energy
*****************************************************************
*
      IMPLICIT NONE
*
```

```
**********************************************************************
C   function MGAUSSMM USES A GAUSS-LEGENDRE QUADRATURE FORMULA TO
C   PERFORM A 5 POINTS PER SUBINTERVAL NUMERICAL INTEGRATION
**********************************************************************
* Integer Variables
*  N            number of subintervals
*  XI           =K-1
*  M,K          loop index
*  J            labels meson type being produced
*  II           number of points in energy array
*  IJ           number of ion types
*  IM           number of meson/muon types
**********************************************************************
      INTEGER N, J, M, K, XI, II, IJ, IM
**********************************************************************
* Real Variables
*  A       lower integration limit
*  B       upper limit
*  SOURCE  function to be integrated
*  E       energy array
*  EPI     energy of meson/muon being produced
```

```
*  EPROJ   energy of projectile-variable of integration
*  FLUX    ion flux
*  FLUXMM  meson/muon flux
*  SUM     result of integration
C   U(I) ARE THE ZEROS OF LEGENDRE POLYNOMIAL OF DEG. 5
C   G(I) ARE THE WEIGHTS SUCH THAT THE INTEGRATION IS EXACT
*****************************************************************
      REAL A,B,SOURCE,EPI,E(II),FLUX(II,IJ),FLUXMM(II,IM),EPROJ,
     $     FOFX,U(5),G(5),FINE,DELTA,UU,SUM
      EXTERNAL SOURCE
*****************************************************************
*---------------------------------------------------------------
*----------------------------start function-----------------------
*---------------------------------------------------------------
*****************************************************************
      SUM=0.
      IF(A.EQ.B) THEN
         MGAUSSMM=0
         RETURN
      END IF
```

```
C   U(I) ARE THE ZEROS OF LEGENDRE POLYNOMIAL OF DEG. 5


      U(1)=.42556283050918

      U(2)=.28330230293537

      U(3)=.160295215850488

      U(4)=.067468316655508

      U(5)=.013046735741414


C   G(I) ARE THE WEIGHTS SUCH THAT THE INTEGRATION IS EXACT


      G(1)=.147762112357376

      G(2)=.13463335965499

      G(3)=.109543181257991

      G(4)=.074725674575290

      G(5)=.033335672154344

      FINE=N

      DELTA=FINE/(B-A)

C   FIND INITIAL VALUE FOR INTERVAL

      DO 40 K=1,N

         XI =K-1

         FINE=A+XI/DELTA
```

```
C     EVALUATE FUNCTION AT 5 POINTS PER INTERVAL

         DO 20 M=1,5
            EPROJ=U(M)/DELTA+FINE
            FOFX=SOURCE(EPI, EPROJ, E, FLUX, FLUXMM, J, II, IJ, IM)

C     ADD TO SUM FOR EACH FUNCTION


            SUM=G(M)*FOFX+SUM
 20      CONTINUE

         DO 30 M=1,5
            EPROJ=(1.-U(M))/DELTA+FINE
            FOFX=SOURCE(EPI, EPROJ, E, FLUX, FLUXMM, J, II, IJ, IM)
            SUM=G(M)*FOFX+SUM
 30      CONTINUE
 40   CONTINUE

      MGAUSSMM=SUM/DELTA
```

```
      RETURN
      END
*********************************************************************
* end function mgaussmm
*********************************************************************
      REAL FUNCTION MMLOSS(EPI, J)
*********************************************************************
* This function calculates total interaction cross section + decay
* length for mesons and muons of type J and of energy EPI
*********************************************************************
*
      IMPLICIT NONE
*
*********************************************************************
* Integer Variables
*  J    is the meson type:  1 is mu+ , 2 mu-, 3 pi+, 4 pi-
*  NAT  number of different types of atoms in medium
*  K    loop variable
*********************************************************************
*
      INTEGER K, J, NAT
```

```
*
*********************************************************************
* Real variables
*  ATRG atomic number of those atoms
*  EPI      is meson energy
*  TOTAL    is meson total interaction cross section with medium
*  PIDECAY  is a function that calculates meson inverse decay length
*  DENSTRG  density of atoms of type K in medium
*  PNST    function returns microscopic total (elastic + inelastic)
*           pion-nucleus interaction cross section
*********************************************************************
      REAL EPI,  TOTAL, PIDECAY, DENSTRG(5), ATRG(5),
     $     INELASTICPIPA, INELASTICPIMA
*********************************************************************
* unused reals for common statement
*********************************************************************
      REAL NLAY,XLAY,DN,ZTRG(5)
*********************************************************************
* set commons
*********************************************************************
      COMMON/TARGET/NLAY,XLAY,DN,NAT,ATRG,ZTRG,DENSTRG
```

```
*********************************************************************
* initialize total
*********************************************************************
*
      TOTAL=0.0
*
*********************************************************************
* calculate macroscopic total (elastic + inelastic) interaction
* cross section
* First consider pions. charge differences between pions are ignored
*********************************************************************
*
      IF (J .EQ. 3) THEN
*
*********************************************************************
* sum micro-cross section over target atoms to get
* a macroscopic cross section
*********************************************************************
*
         DO 10 K=1,NAT
*
```

```
************************************************************************
* calculation based on scaling from pi+ p collisions to pi+ A
************************************************************************
*
            TOTAL= TOTAL + DENSTRG(K)*INELASTICPIPA(ATRG(K),EPI)
*
************************************************************************
* end sum over target atoms
************************************************************************
*
 10      CONTINUE
*
************************************************************************
*
      ELSE IF (J .EQ. 4) THEN
*
************************************************************************
* sum micro-cross section over target atoms
************************************************************************
*
         DO 20 K=1,NAT
```

```
*
**********************************************************************
* calculation based on scaling from pi p collisions to pi A
**********************************************************************
*
            TOTAL= TOTAL + DENSTRG(K)*INELASTICPIMA(ATRG(K),EPI)
*
**********************************************************************
* end sum over target atoms
**********************************************************************
*
 20      CONTINUE
*
**********************************************************************
* set interaction cross section to zero for muons
**********************************************************************
*
         ELSE
            TOTAL=0.0
*
**********************************************************************
```

```
*
         END IF
*
*********************************************************************
* add in losses due to decay
*********************************************************************
*
         MMLOSS=PIDECAY(EPI, J)+TOTAL
*
*********************************************************************
* end
*********************************************************************
         RETURN
         END
*********************************************************************
* end function mmloss
*********************************************************************
      SUBROUTINE MULIMITS(EMU, THRESHOLD, UPPERLIM)
*********************************************************************
* this subroutine calculates integration limits for muon production
* from pion decay
```

```
************************************************************************
*
      IMPLICIT NONE
*
************************************************************************
* Real Variables
* emu           is the muon energy
* emumev        is the muon energy in units of MeV
* pmu           is the muon momentum in umits of Mev
* threshold     is the minimum energy a pion needs to produce a muon
*                   of energy emu
* upperlim      is the maximum energy a pion can have and still
*                   produce a muon of energy emu
* pstar         momentum of a produced muon in the pion rest frame
* estar         total energy of a produced muon in the pion
*                   rest frame
* mmu           muon mass
* amu           muon mass in atomic mass units
* mpi           pion mass
* api           pion mass in atomic mass units
************************************************************************
```

```
      REAL EMU,THRESHOLD,UPPERLIM,EMUMEV,PMUMEV,
     $     PSTAR,ESTAR,MMU,MPI,API,AMU
*********************************************************************
*-------------------------------------------------------------------
*------------------START SUBROUTINE MULIMITS-----------------------
*-------------------------------------------------------------------
*********************************************************************
* set masses
*********************************************************************
*
      API= 0.14983
      AMU= 0.11342
      MPI= 139.570
      MMU= 105.658
*
*********************************************************************
* tranform muon  energy to units of MeV
*********************************************************************
*
      EMUMEV=EMU*AMU
*
```

```
*********************************************************************
* calculate limits
*********************************************************************
*
      ESTAR=(MPI**2.0+MMU**2.0)/(2.0*MPI)
      PSTAR=SQRT(ESTAR**2.0-MMU**2.0)
      PMUMEV=SQRT((EMUMEV+MMU)**2.0 - MMU**2.0)
      THRESHOLD=(ESTAR*(EMUMEV+MMU)-PSTAR*PMUMEV)*MPI/(MMU**2.0)
     $     - MPI
      UPPERLIM=(ESTAR*(EMUMEV+MMU)+PSTAR*PMUMEV)*MPI/(MMU**2.0)
     $     - MPI
*
*********************************************************************
* convert limits to MeV/amu and prevent mistakes due to round off
*********************************************************************
*
*      IF (THRESHOLD .LT. 0.0) THRESHOLD=0.0
      THRESHOLD=THRESHOLD/API
      UPPERLIM=UPPERLIM/API
*
*********************************************************************
```

```
* end subroutine mulimits
*****************************************************************
      RETURN
      END
*****************************************************************
      REAL FUNCTION MUPRODPI(EPI, EMU)
*****************************************************************
* This function returns macroscopic cross section for prouction
* muons from decay of like charged pions
*****************************************************************
*
      IMPLICIT NONE
*
*****************************************************************
* Real variables
*  EPI   energy of projectile pion that is decaying in MeV/amu
*  EMU     energy of muon being created in MeV/amu
*  API     mass of pion in MeV/amu
*  AMU     mass of muon in MeV/amu
*  ESTAR   total energy of muon in pion rest frame
*  PSTAR   momentum of muon in pion rest frame
```

```
*  GAMMA   lorentz factor of pion relative to medium
*  BETA    speed/c of pion relative to medium
*  UPPER   upper limit to energy of muon produced from pion of
*          energy EPROJ
*  LOWER   lower limit
*  PIDECAY function that returns inverse decay length of pion
**********************************************************************
      REAL EPI, EMU,EPIMEV, EMUMEV, PSTAR, GAMMA, BETA, ESTAR,
     $     UPPER, LOWER, PIDECAY, API, AMU, MPI, MMU
**********************************************************************
*--------------------------------------------------------------------
*----------------------------start function muprodpi---------------
*--------------------------------------------------------------------
**********************************************************************
* set masses
**********************************************************************
*
      API= 0.14983
      AMU= 0.11342
      MPI= 139.570
      MMU= 105.658
```

```
*
**********************************************************************
* tranform muon  and pion energy to units of MeV
**********************************************************************
*
      EMUMEV=EMU*AMU
      EPIMEV=EPI*API
*
**********************************************************************
* calculate kinematic quantities
**********************************************************************
*
      GAMMA= (EPIMEV + MPI)/MPI
      BETA = (1.0 - 1.0/(GAMMA**2.0))**0.5
      ESTAR=(MPI**2.0+MMU**2.0)/(2.0*MPI)
      PSTAR=SQRT(ESTAR**2.0-MMU**2.0)
      UPPER=GAMMA*(ESTAR+BETA*PSTAR)-MMU
      LOWER=GAMMA*(ESTAR-BETA*PSTAR)-MMU
*
**********************************************************************
*     calculate cross section
```

```
************************************************************************
* convert pstar to Mev/amu
      PSTAR=PSTAR/AMU
************************************************************************
*
      IF ((EMUMEV .LE. UPPER) .AND. (EMUMEV .GE. LOWER)) THEN
         MUPRODPI=PIDECAY(EPI,3)/(2.0*BETA*GAMMA*PSTAR)
      ELSE
         MUPRODPI=0.0
      END IF
*
************************************************************************
* end function muprodpi
************************************************************************
      RETURN
      END
************************************************************************
************************************************************************
      subroutine output(psi,psimm,st,et)
      implicit none
      integer II,IJ,IM,ie,j
```

```
      parameter(II=45,IJ=59,IM=4)
      real psi(ii,ij), psimm(ii,im), st(ii), et(ii),
     $     temp1, anumm, anu

      open(12,file='mmflux.dat',status='old',position='append')
      open(13,file='percent_flux.dat',status='old',position='append')

      DO 70 IE=1,II
         write(12,120) et(IE), psimm(ie,1)/(anumm(1)*st(ie)),
     $        psimm(ie,2)/(anumm(2)*st(ie)),
     $        psimm(ie,3)/(anumm(3)*st(ie)),
     $        psimm(ie,4)/(anumm(4)*st(ie)),
     $        psimm(ie,1)/(anumm(1)*st(ie))+
     $        psimm(ie,2)/(anumm(2)*st(ie))+
     $        psimm(ie,3)/(anumm(3)*st(ie))+
     $        psimm(ie,4)/(anumm(4)*st(ie))
120      Format(E11.4, E11.4, E11.4, E11.4, E11.4, E11.4)

         temp1=0.0
         do 111 j=1,2
            temp1= temp1+psi(ie,J)
```

```
111     continue
        do 80 J=3,IJ
           temp1= temp1+psi(ie,J)/anu(J)
80      continue

        do 90 J=1,IM
           temp1=temp1+psimm(ie,J)/anumm(J)
90      continue

        write(13,130) et(IE), 100.*psimm(ie,1)/(anumm(1)*temp1),
    $        100.* psimm(ie,2)/(anumm(2)*temp1),
    $        100.* psimm(ie,3)/(anumm(3)*temp1),
    $        100.* psimm(ie,4)/(anumm(4)*temp1),
    $        100.* psimm(ie,1)/(anumm(1)*temp1) +
    $        100.* psimm(ie,2)/(anumm(2)*temp1)+
    $        100.* psimm(ie,3)/(anumm(3)*temp1)+
    $        100.* psimm(ie,4)/(anumm(4)*temp1)

130     Format(E11.4, E11.4, E11.4, E11.4, E11.4, E11.4)

70   continue
```

```
      close(12)
      close(13)
      return
      end
*********************************************************************
* end subroutine output
*********************************************************************

      SUBROUTINE PGTMM(II, IJ, E, R, ST, PSI, PSIMM,
     $     H, QMOLD)
*********************************************************************
* This subprogram propagates the meson and muon components of
* radiation a distance H.
*********************************************************************
*
      IMPLICIT NONE
*
*********************************************************************
* Integer Variables
*  II  is the number of range and energy bins
*  IJ  is the number of ions
```

```
*  IM  is the number of mesons and muons being transported
*         1 is mu+ , 2 mu-, 3 pi+, 4 pi-
*  I,J are loop indicies
*  IE  is loop index for energy and range bins
************************************************************************
      INTEGER II, IJ, IM, I, J, IE
      PARAMETER (IM=4)
************************************************************************
* Real Variables
*  r        is the range array for protons
*  e        is energy in MeV/Amu
*  flux     are the fluxes of ions
*  psi      are the fluxes of ions multiplied by stopping power
*  fluxmm   are the fluxes of mesons and muons
*  psimm    fluxes of mesons and muons multiplied by stopping power
*  mmsource is the meson/muon source term
*  qm       array of mmsources mutiplied by meson stopping power
*  qmold    is qm from the previous iteration
*  deltaqm  is the spatial derivitive of qm
*  st       is proton stopping power
*  scale    stopping power scaling factor needed for source terms
```

```
*  aloss    total interaction cross section + inverse decay length
*  qm       is the pion and meson source terms
*  H        is the stepsize
*****************************************************************
      REAL E(II), R(II), PSI(II, IJ), FLUX(II,IJ), PSIMM(II,IM),
     $     FLUXMM(II,IM), QMOLD(II, IM), QM(II,IM), ST(II),
     $     MMSOURCE, DELTAQM, SCALE, H, TEMP1,
     $     ALOSS,  TEMP2, THRESHOLD, UPPERLIM
*****************************************************************
* Functions Called
*  anumm    calculates stopping power scaling factors for mesons
*  anu      calculates stopping power scaling factors for ions
*  mmloss   calculates total interaction cross section + decay length
*  phi      interpolates flux arrays; in original hzetrn
*  source   calculates meson sources at a specific energy
*  mgaussmm integrates source over energy
*  mulimts  subroutine caculates limits of integration
*           for muon production
*****************************************************************
      REAL ANUMM, ANU, MMLOSS, PHI, SOURCE, MGAUSSMM
      EXTERNAL SOURCE
```

```
*********************************************************************
*-------------------------------------------------------------------
*--------------------------START PROPAGATION-------------------------
*-------------------------------------------------------------------
*********************************************************************
* calculate fluxes for source term by dividing by stopping power
*********************************************************************
*
      DO 10 IE=1,II
         DO 20 J=1,IM
            FLUXMM(IE,J)=PSIMM(IE,J)/(ANUMM(J)*ST(IE))
 20      CONTINUE
         DO 140 J=1,2
            FLUX(IE,J)=PSI(IE,J)/ST(IE)
 140     CONTINUE
         DO 100 J=3,IJ
            FLUX(IE,J)=PSI(IE,J)/(ANU(J)*ST(IE))
 100     CONTINUE
 10   CONTINUE
*
*********************************************************************
```

```
* loop for meson and muon particle types
**********************************************************************
*
      DO 30 J=1,IM
*
**********************************************************************
* calculate stopping power scaling factor
**********************************************************************
*
         SCALE=ANUMM(J)
*
**********************************************************************
* loop for range grid
**********************************************************************
*
         DO 40 IE=1,II
*
**********************************************************************
* calculate rate of particle loss at energy IE
**********************************************************************
*
```

```
            ALOSS= MMLOSS(E(IE), J)
*
**********************************************************************
* calculate integration limits for particle production
* Keep them within the energy range being transperted
**********************************************************************
*
            IF ((J.EQ.1).OR.(J.EQ.2)) THEN
               CALL MULIMITS(E(IE),THRESHOLD,UPPERLIM)
               IF (THRESHOLD .LT. E(1)) THRESHOLD=E(1)
               IF (UPPERLIM .GT. E(II)) UPPERLIM=E(II)
            ELSE
*               IF (E(IE) .GT. 290.0) THEN
*                  THRESHOLD=E(IE)
*               ELSE
                  THRESHOLD=290.0
*               END IF
               UPPERLIM=E(II)
            END IF
*            print*, threshold, upperlim, E(IE)
*
```

```
*********************************************************************
* calculate source terms
*********************************************************************
*
            MMSOURCE=MGAUSSMM(THRESHOLD,UPPERLIM,6,SOURCE,J,
     $           E(IE),E,FLUX,FLUXMM,II,IJ,IM)
            QM(IE,J)=SCALE * ST(IE)*MMSOURCE
            DELTAQM=(QM(IE,J)-QMOLD(IE,J))/H
*
*********************************************************************
* calculate new psimm
* note that  (h/2)*(1.0+0.66666666*a*h +((a*h)**2.0)*exp(ah)/12.0
*            = exp(ah)*(h/a-1/a**2) +1/a**2
*********************************************************************
*
            TEMP1=QM(IE,J)*(EXP(ALOSS*H)-1.0)/ALOSS
            TEMP2=(H**2.0/2.0)*(1.0+0.66666666*ALOSS*H
     $           +((ALOSS*H)**2.0)*exp(ALOSS*H)/12.0)
            PSIMM(IE,J)=EXP(-H*ALOSS)*
     $           (PHI(R(II)+1.0,II,R,PSIMM(1,J),R(IE) + SCALE*H)
     $           +TEMP1 + DELTAQM*TEMP2)
```

```
*
**********************************************************************
* end loop over range
**********************************************************************
*
 40      CONTINUE
*
**********************************************************************
* end loop over meson type
**********************************************************************
*
 30   CONTINUE
*
**********************************************************************
* set source term for use in next iteration
**********************************************************************
*
      DO 50 J=1,IM
         DO 60 IE=1,II
            QMOLD(IE,J)=QM(IE,J)
 60      CONTINUE
```

```
 50   CONTINUE
*
**********************************************************************
* end subroutine pgtmm
**********************************************************************
      RETURN
      END
**********************************************************************
**********************************************************************
      REAL FUNCTION PIDECAY(EPI, J)
**********************************************************************
* This function returns the inverse decay length of pions/muons of
* type J and energy EPI.
**********************************************************************
*
      IMPLICIT NONE
*
**********************************************************************
* Integer Variables
*  IM  number of mesons and muons
*  J   indexes particle type 1, 2, 3 or 4 correspond to
```

```
*      mu+, mu-, pi+, and pi-
*****************************************************************
      INTEGER  IM, J
      PARAMETER(IM=4)
*****************************************************************
* Real Variables
*   EPI   is the energy in MeV/amu
*   EMEV  is the energy in MeV
*   TIME  is the lifetime in rest frame in seconds
*   C     is  the speed of light in cm/s
*   MPI   are particle masses in MeV/c^2
*   API   is mass in amu
*   GAMMA is the time dilation factor.
*   BETA  is the velocity/c
*   DN    is the target density in gm/cm^3
*****************************************************************
      REAL EPI, TIME(IM), C, MPI(IM), GAMMA, BETA, API(IM), EMEV, DN
      PARAMETER (C=2.9979245800E10)
      DATA TIME /2.19703E-6, 2.19703E-6, 2.6033E-8, 2.6033E-8/
      DATA MPI /105.658, 105.658, 139.57, 139.57/
      DATA API /0.11342, 0.11342, 0.14983, 0.14983/
```

```
*********************************************************************
* unused reals for common statement
*********************************************************************
      REAL NLAY,XLAY,NAT,ZTRG(5),ATRG(5),DENSTRG(5)
*********************************************************************
* set commons
*********************************************************************
      COMMON/TARGET/NLAY,XLAY,DN,NAT,ATRG,ZTRG,DENSTRG
*********************************************************************
* trans form energy from MeV/amu to MeV
*********************************************************************
*
      EMEV=EPI*API(J)
*
*********************************************************************
* calculate inverse decay length
*********************************************************************
*
      GAMMA= (EMEV + MPI(J))/MPI(J)
      BETA = (1.0 - 1.0/(GAMMA**2.0))**0.5
      PIDECAY=1.0/(BETA*C*TIME(J)*GAMMA*DN)
```

```
*
**********************************************************************
* end function pidecay
**********************************************************************
      RETURN
      END
**********************************************************************
**********************************************************************
      REAL FUNCTION PIMAPIM(EPROJ,EPI,AT)
**********************************************************************
* this function that returns microscopic spectral distribution for
* the reaction pi- + A -> pi- + x
**********************************************************************
*
      IMPLICIT NONE
*
**********************************************************************
* Real Variables
*  EPROJ   kinetic energy of projectile pi+
*  EPI     kinetic energy of produced   pi+
*  PIMINUS function returns spectral distribution for reaction
```

```
*          proton + proton -> pi- +x
*  ALPHA   scales collisions from target nucleon to nucleus
*  AT    atomic number of target
**********************************************************************
      REAL EPROJ, EPI, PIMINUS, ALPHA, AT
**********************************************************************
*----------------------------------------------------------------------
*-------------------------start function PIMAPIM----------------------
*----------------------------------------------------------------------
**********************************************************************
* set alpha
**********************************************************************
*
      ALPHA = 0.77
*
**********************************************************************
* calculate cross section
* this formulae assumes pion projectiles act like protons
**********************************************************************
*
      PIMAPIM=0.62*PIMINUS(EPROJ,EPI)*AT**ALPHA
```

```
*
**********************************************************************
* end function pimapim
**********************************************************************
      RETURN
      END
**********************************************************************
**********************************************************************
      REAL FUNCTION PIMAPIP(EPROJ,EPI,AT)
**********************************************************************
* this function that returns microscopic spectral distribution for
* the reaction pi- + A -> pi+ + x
**********************************************************************
*
      IMPLICIT NONE
*
**********************************************************************
* Real Variables
*  EPROJ  kinetic energy of projectile pi-
*  EPI    kinetic energy of produced   pi+
*  PIPLUS function returns spectral distribution for reaction
```

```
*         proton + proton -> pi+ +x
*  ALPHA  scales collisions from target nucleon to nucleus
*  A    atomic number of target
**********************************************************************
      REAL EPROJ, EPI, PIPLUS, ALPHA, AT
**********************************************************************
*----------------------------------------------------------------------
*-------------------------start function PIMAPIP----------------------
*----------------------------------------------------------------------
**********************************************************************
* set alpha
**********************************************************************
*
      ALPHA = 0.77
*
**********************************************************************
* calculate cross section
* this formulae assumes pion projectiles act like protons
**********************************************************************
*
      PIMAPIP=0.62*PIPLUS(EPROJ,EPI)*AT**ALPHA
```

```
*
**********************************************************************
* end function pimapip
**********************************************************************
      RETURN
      END
**********************************************************************
**********************************************************************

      REAL FUNCTION PIMINUS(EPR, E)
**********************************************************************
* This function returns microscopic spectral distribution for
* negative pion production from proton-proton collisions using a
* parameterization. The formula is split into three different
* energy regions
**********************************************************************
*
      IMPLICIT NONE
*
**********************************************************************
```

```
* Real Variables
*  G and H parameters of parametrization
*  EPR     energy of projectile proton in MeV
*  E       energy of pion in MeV
*  EPROJ   energy of projectile proton in GeV
*  EPION   energy of pion in GeV
**********************************************************************
      REAL TEMP1, TEMP2, G(14), H(17), EPROJ, EPI, EPR, E, API
**********************************************************************
*---------------------------------------------------------------------
*---------------------------------start function PIMINUS-------------
*---------------------------------------------------------------------
**********************************************************************
* set parameters
**********************************************************************
*
      DATA G /1.06E-9,-2.8,3.7E-8,-1.89,22.3,-1.5,-30.5,0.0938,-24.6,
     $     0.0313,0.25,2.5E6,7.96,-49.5/
      DATA H /2.39E-10,-2.8,1.14E-8,-2.3,22.3,-2.23,-31.3,0.0938,
     $     -24.9,0.0313,2.5E6,0.25,60322,1.1,-65.9,-9.39,-1.25/
      API=0.14983
```

```
*
**********************************************************************
* change energy from MeV/amu to GeV, because the parametrization
* assumes energy is in units of GeV.  A proton has an atomic number
* of one so the amu part does not need to be converted.
**********************************************************************
*
      EPI= E*API/1000.0
      EPROJ=EPR/1000.0
*
**********************************************************************
* if pion energy too low set it to prevent overflow
**********************************************************************
*
      IF (EPI .LT. 0.000001) THEN
         EPI=0.000001
      END IF
*
**********************************************************************
* calculate parameterization which is broken up into 3 energy ranges.
**********************************************************************
```

```
*
      IF (EPROJ .GE. .29 .AND. EPROJ .LE. 2.0) THEN
         TEMP2 = G(1)*EPI**G(2) + G(3)*EPROJ**G(4)
         TEMP1=EXP(G(5) + G(6)/SQRT(EPROJ) + G(7)*EPI**G(8) +
     $        G(9)*EPI**G(10))
         PIMINUS = EPI**G(11)*(G(12)*TEMP1/TEMP2 +
     $        G(13)*EXP( G(14)*SQRT(EPI)))


      ELSE IF (EPROJ .GT. 2.0) THEN

         TEMP2 = H(1)*EPI**H(2) +H(3)*EPROJ**H(4)
         TEMP1 = EXP(H(5) + H(6)/SQRT(EPROJ) + H(7)*EPI**H(8)
     $        +H(9)*EPI**H(10))
         PIMINUS = H(11)*EPI**H(12)*TEMP1/TEMP2 +  H(13)
     $        *EPI**H(14)*EXP(H(15)*SQRT(EPI)+H(16)*EPROJ**H(17))


      ELSE
         PIMINUS =0.0
         RETURN
      END IF
```

```
*
**********************************************************************
* change spectral distribution from mb/GeV to cm^2*amu/MeV
**********************************************************************
*
      PIMINUS=PIMINUS*API*1.0E-30
*
**********************************************************************
* end function piminus
**********************************************************************
      RETURN
      END
**********************************************************************
**********************************************************************

      REAL FUNCTION PIPAPIM(EPROJ,EPI,AT)
**********************************************************************
* this function that returns microsopic spectral distributiuon for
* the reaction pi+ + A -> pi- + x
**********************************************************************
*
```

```
      IMPLICIT NONE
*
*************************************************************************
* Real Variables
*  EPROJ   kinetic energy of projectile pi+
*  EPI     kinetic energy of produced   pi+
*  PIMINUS function returns spectral distribution for reaction
*          proton + proton -> pi- +x
*  ALPHA   scales collsions from target nucleon to nucleus
*  AT    atomic number of target
*************************************************************************
      REAL EPROJ, EPI, PIMINUS, ALPHA, AT
*************************************************************************
*-----------------------------------------------------------------------
*-------------------------start function PIPAPIM----------------------
*-----------------------------------------------------------------------
*************************************************************************
* set alpha
*************************************************************************
*
      ALPHA = 0.77
```

```
*
**********************************************************************
* calculate cross section
* this formulae assumes pion projectiles act like protons
**********************************************************************
*
      PIPAPIM=0.62*PIMINUS(EPROJ,EPI)*AT**ALPHA
*
**********************************************************************
* end function pipapim
**********************************************************************
      RETURN
      END
**********************************************************************
**********************************************************************

      REAL FUNCTION PIPAPIP(EPROJ,EPI,AT)
**********************************************************************
* this function that returns microsopic spectral distributiuon for
* the reaction pi+ + A -> pi+ + x
**********************************************************************
```

```
*
      IMPLICIT NONE
*
*********************************************************************
* Real Variables
*  EPROJ  kinetic energy of projectile pi+
*  EPI    kinetic energy of produced   pi+
*  PIPLUS function returns spectral distribution for reaction
*         proton + proton -> pi+ +x
*  ALPHA  scales collsions from target nucleon to nucleus
*  AT    atomic number of target
*********************************************************************
      REAL EPROJ, EPI, PIPLUS, ALPHA, AT
*********************************************************************
*--------------------------------------------------------------------
*-------------------------start function PIAPIP----------------------
*--------------------------------------------------------------------
*********************************************************************
* set alpha
*********************************************************************
*
```

```
      ALPHA = 0.77
*
**********************************************************************
* calculate cross section
* this formulae assumes pion projectiles act like protons
**********************************************************************
*
      PIPAPIP=0.62*PIPLUS(EPROJ,EPI)*AT**ALPHA
*
**********************************************************************
* end function pipapip
**********************************************************************
      RETURN
      END
**********************************************************************
**********************************************************************
      REAL FUNCTION PIPLUS(EPR, EPI)
**********************************************************************
* This function returns microscopic spectral distribution for
* positive pion production from proton-proton collisions using a
* parameterization. The formula is split into three different
```

```
* energy regions.
*********************************************************************
*
      IMPLICIT NONE
*
*********************************************************************
* Real Variables
*  C and D parameters of parameterization
*  EPR     energy of projectile proton in MeV
*  EPI     energy of pion in MeV
*  EPROJ   energy of projectile proton in GeV
*  EPION   energy of pion in GeV
*********************************************************************
      REAL TEMP1, TEMP2, C(20), D(17), EPROJ, EPION, EPI, EPR, API
*********************************************************************
*-------------------------------------------------------------------
*---------------------------------start function PIPLUS--------------
*-------------------------------------------------------------------
*********************************************************************
* set parameters
*********************************************************************
```

```
*
      DATA C /2.2E-8,-2.7,4.22E-7,-1.88,22.3,1.98,-0.28,-1.75,
     $     -29.4,0.0938,-24.4,0.0312,0.0389,1.78,2.5E6,0.25,
     $     976,2.3,-46,-0.989/
      DATA D /4.5E-11,-2.98,1.18E-9,-2.55,22.3,-0.765,-35.3,0.0938,
     $     -22.5,0.0313,2.56E6,0.25,60322,1.18,-72.2,0.9421,0.1/

      API=0.14983
*
**********************************************************************
* change energy from MeV/amu to GeV. the parameterization assumes
* energy is in units of GeV. Note that a proton has an atomic number
* of one so the amu part does not need to be converted.
**********************************************************************
*
      EPION=EPI*API/1000.0
      EPROJ=EPR/1000.0
*
**********************************************************************
* if pion energy too low set it to prevent overflow
**********************************************************************
```

```
*
      IF (EPION .LT. 0.000001) THEN
         EPION = 0.000001
      END IF
*
**********************************************************************
* calculate parameterization which is broken up into 3 energy ranges.
**********************************************************************
*
      IF ((EPROJ .GE. .29) .AND. (EPROJ .LE. 2.0)) THEN
         TEMP2 = C(1)*EPION**C(2) + C(3)*EPROJ**C(4)
         TEMP1=EXP(C(5)+C(6)/SQRT(EPROJ)+C(7)*EPROJ**C(8)
     $        +C(9)*EPION**C(10)+C(11)*EPION**C(12)*EPROJ**C(13)
     $        +C(14)*LOG(EPROJ))
         PIPLUS = C(15)*EPION**C(16)*TEMP1/TEMP2 + C(17)*
     $        EPION**C(18)*EXP(C(19)*SQRT(EPION)
     $        + C(20)*SQRT(EPROJ))

      ELSE IF (EPROJ .GT. 2.0) THEN

         TEMP2 = D(1)*EPION**D(2) +D(3)*EPROJ**D(4)
```

```
          TEMP1 = EXP(D(5) + D(6)/SQRT(EPROJ) + D(7)*EPION**D(8)
     $         +D(9)*EPION**D(10))
          PIPLUS = D(11)*EPION**D(12)*TEMP1/TEMP2 +
     $         D(13)*EPION**D(14)*EXP(D(15)*SQRT(EPION)
     $         +D(16)*EPROJ**D(17))

       ELSE
          PIPLUS =0.0
       END IF
*
**********************************************************************
* change spectral distribution from mb/GeV to cm^2*amu/MeV
* 1mb=10^-27 cm^2
**********************************************************************
*
       PIPLUS=PIPLUS*1.0E-30*API
*
**********************************************************************
* end function piplus
**********************************************************************
       RETURN
```

```
      END
*********************************************************************
*********************************************************************
      REAL FUNCTION SOURCE(EPI, EPROJ, E, FLUX, FLUXMM, J, II,
     $     IJ, IM)
*********************************************************************
* This function calculates meson sources at specific energies.
* It is integrated over that energy by mgaussmm. After this
* function is integrated by mgaussmm, it is the function
* q_j(x,r_i) described in the text.
*********************************************************************
*
      IMPLICIT NONE
*
*********************************************************************
* Integer Variables
*  II     number of range grid points
*  IJ     total number of ions
*  IM     total number of mesons/muons
*  P      index for projectiles
*  J      type of meson being produced
```

```
************************************************************************
      INTEGER II, IJ, P, J, IM
************************************************************************
* Real Variables
*  E        energy grid
*  FLUX     ion flux array
*  FLUXMM   meson flux array
*  EPI      energy of meson being produced
*  EPROJ    energy of projectile
*  FLUENCE  interpolated value of flux
*  AAMMCS   function that returns macroscopic cross section for
*           production of mesons from collisions of nuclei with
*           the medium
*  MAMMCS   function that returns macroscopic cross section for
*           production of mesons from collisions of mesons with
*           the medium
*  MUPRODPI function that returns macroscopic cross section for
*           production muons from decay of like charged pions
*  PHI      function that interpolates fluxes
*  TEMPSOURCE
************************************************************************
```

```
      REAL E(II), FLUX(II,IJ), FLUXMM(II,IM), EPI,EPROJ,FLUENCE,
     $     AAMMCS,  MAMMCS, PHI, MUPRODPI, TEMPSOURCE
*********************************************************************
*-------------------------------------------------------------------
*-------------------------start function----------------------------
*-------------------------------------------------------------------
*********************************************************************
* initialize source
*********************************************************************
*
      TEMPSOURCE=0.0
*
*********************************************************************
* Ignore muon production from collisions
*********************************************************************
*
      IF (J .GT. 2) THEN
*
*********************************************************************
*  sum over projectile nucleons
*********************************************************************
```

```
*
         DO 50 P=1,IJ
*
**********************************************************************
*  calculate projectile flux at energy eproj.
**********************************************************************
*
            FLUENCE=PHI(E(II)+1.0,II,E,FLUX(1,P),EPROJ)
            TEMPSOURCE=TEMPSOURCE+FLUENCE*AAMMCS(EPROJ,EPI,P,J)
*
**********************************************************************
* end sum over projectile nucleons
**********************************************************************
*
 50      CONTINUE
*
**********************************************************************
* sum over meson projectiles. no muons included
**********************************************************************
*
         DO 60 P=3,IM
```

```
*
**********************************************************************
* calculate projectile flux at energy eproj.
**********************************************************************
*
            FLUENCE=PHI(E(II)+1.0,II,E,FLUXMM(1,P),EPROJ)
            TEMPSOURCE=TEMPSOURCE+FLUENCE*MAMMCS(EPROJ,EPI,P,J)
*
**********************************************************************
* end sum over meson projectiles
**********************************************************************
*
 60      CONTINUE
*
**********************************************************************
*     now do muon production
*     muons are being produced only from pion decay.
*     kaon are not yet included.
**********************************************************************
*
      ELSE
```

```
          FLUENCE=PHI(E(II)+1.0,II,E,FLUXMM(1,J+2),EPROJ)
          TEMPSOURCE=TEMPSOURCE+FLUENCE*MUPRODPI(EPROJ, EPI)
*
**********************************************************************
* end function source
**********************************************************************
       ENDIF
       SOURCE=TEMPSOURCE
       RETURN
       END
**********************************************************************
```

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14. ABSTRACT

A safe and efficient exploration of space requires an understanding of space radiations, so that human life and sensitive equipment can be protected. On the way to these sensitive sites, the radiation fields are modified in both quality and quantity. Many of these modifications are thought to be due to the production of pions and muons in the interactions between the radiation and intervening matter. A method used to predict the effects of the presence of these particles on the transport of radiation through materials is developed. This method was then used to develop software, which was used to calculate the fluxes of pions and muons after the transport of a cosmic ray spectrum through aluminum and water. Software descriptions are given in the appendices.

15. SUBJECT TERMS

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