

# Calculation of the Coulomb Fission Cross Sections for $\mathrm{Pb}-\mathrm{Pb}$ and $\mathrm{Bi}-\mathrm{Pb}$ Interactions at 158 A GeV 

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#### Abstract

The Weizsäcker-Williams (WW) method of virtual quanta is used to make approximate cross section calculations for peripheral relativistic heavy-ion collisions. We calculated the Coulomb fission cross sections for projectile ions of ${ }^{208} \mathrm{~Pb}$ and ${ }^{209} \mathrm{Bi}$ with energies of 158 A GeV interacting with a ${ }^{208} \mathrm{~Pb}$ target. We also calculated the electromagnetic absorption cross section for a ${ }^{208} \mathrm{~Pb}$ ion interacting as described. For comparison we use both the full $W W$ method and a standard approximate $W W$ method. The approximate $W W$ method resulted in larger cross sections compared with the more accurate full WW method.


## 1. Introduction

An important process neglected in the transport codes FLUKA (refs. 1 and 2) and MCNPX (ref. 3) is nuclear electromagnetic dissociation (refs. 4-8). When two nuclei collide, with an impact parameter less than or equal to the sum of the radii, they break up due to the strong forces. However, if the impact parameter is greater than the sum of the nuclear radii then breakup can occur via the electromagnetic (EM) interaction. This is especially important for few-nucleon removal (including neutrons) and for medium nuclei such as Aluminum or Iron. The few-nucleon EM removal cross sections can be larger than strong interactions cross sections (ref. 5). The EM interaction also leads to large fission cross sections (refs. 6 and 7) and double EM processes lead to a copious amount of electron-positron production (ref. 4) with cross sections in the kilobarn region. FLUKA and MCNPX include none of these processes.

The transport code HZETRN uses the nuclear fragmentation code NUCFRG (ref. 9 ), which contains a description of high energy nucleus-nucleus collisions in terms of a modified abrasion-ablation model. An advantage of NUCFRG is that it contains code for nuclear break up due to both the strong and electromagnetic interactions (ref. 8).

None of the above three transport codes include electromagnetic dissociation cross sections leading to few-nucleon removal or fission. The present paper continues the study of relativistic Coulomb fission (refs. 6 and 7) with a view to including Coulomb fission cross sections in future versions of transport codes.

There are two basic mechanisms that can induce fission for relativistic heavy-ion collisions. The predominant mechanism is nuclear fission. This is the case when the perpendicular distance between ion centers as they pass each other, the impact parameter $b$, is less than the sum of the ions' radii. Another mechanism is due to the electromagnetic interactions between the ions. This is the case when $b$ becomes greater than the sum of the radii of the two ions and it is called Coulomb fission (ref. 7). The objective of this study is to calculate the Coulomb fission cross sections for ${ }^{208} \mathrm{~Pb}$ and ${ }^{209} \mathrm{Bi}$ ions at

158 A GeV interacting with a stationary ${ }^{208} \mathrm{~Pb}$ target using the full Weizsäcker-Williams (WW) method and an approximate WW method. We will verify the Coulomb fission and the electromagnetic absorption cross section calculations of Abreu et al. (ref. 10) based on the approximate WW method that they used. We will be using their notations and conventions.

## 2. The Equivalent Photon Approximation

The Equivalent Photon Approximation, also known as the Weizsäcker-Williams Method of Virtual Quanta (ref. 11), is a classical computational scheme based upon a plane wave approximation to radiation that is Lorentz contracted in the direction of motion and concentrated normal to that direction. The original idea came from Fermi in 1924 (ref. 12). It was then extended independently by Williams in 1934 (ref. 13) and Weizsäcker in 1934 (ref. 14). Consider the target ion, $B$, from the reference frame of the projectile ion, $A$. The stationary target has an apparent velocity toward the projectile ion. As the speed of closure approaches that of light, the target's now electromagnetic (motional induced magnetic) field can be modeled as plane wave radiation or as an equivalent swarm of virtual photons. We view the collision in cylindrical coordinates centered on the target where the projectile trajectory is parallel to the axis but offset by the impact parameter, $b$. The target ion's electric field is Lorentz contracted in the longitudinal direction along the axis and concentrated with respect to the circular radial direction. Accordingly, the photon radiation from the target can be replaced by two radiation pulses, $P_{1}$ and $P_{2}$. Pulse $P_{1}$ has the concentrated electric field perpendicular to the motion and pulse $P_{2}$ has the longitudinal electric field.

The Weizsäcker-Williams method uses a function, $N(E)$, that represents the number of virtual photons with energy $E$ in the radiation pulses per unit energy. This number spectrum can be written as an integral over the impact parameter plane as

$$
\begin{equation*}
N(E)=\int_{b_{\min }}^{\infty} N(E, b) 2 \pi b d b \tag{1}
\end{equation*}
$$

where $b_{\min }$ is the minimum impact parameter. The argument in equation (1) can be written in terms of the contributions from the two pulses $P_{1}$ and $P_{2}$ for a point ion (ref. 11) as

$$
\begin{equation*}
N(E, b)=\frac{\alpha Z^{2} \xi^{2}}{\pi^{2} \beta^{2} b^{2} E}\left[K_{1}(\xi)^{2}+\frac{1}{\gamma^{2}} K_{0}(\xi)^{2}\right] \tag{2}
\end{equation*}
$$

where $\alpha=\frac{e^{2}}{\hbar c}$ (the QED fine structure constant), $\xi=\frac{E b}{\gamma \beta c \hbar}, \beta=\frac{v}{c}, \gamma$ is the Lorentz contraction factor, $Z$ is the number of positive charges in the ion, and $K_{n}(\xi)$ are modified Bessel functions of the second kind of order $n$. Equation (2) gives the number of virtual photons (quanta) with energy $E=\hbar \omega$ at transverse position $b$ from the target ion per
unit energy per unit area as seen by the projectile ion traveling at speed $v$. Note that as the speed $v \rightarrow c$, then $\frac{1}{\gamma} \rightarrow 0$. In this limit the second term in equation (2) can be dropped.

The integration of equation (1) can be performed by using the modified Bessel differential equation with relations for the first derivatives of the modified Bessel functions of the second kind. The resulting expression becomes

$$
\begin{equation*}
N(E)=\frac{2 \alpha Z^{2}}{\pi \beta^{2} E}\left\{x K_{0}(x) K_{1}(x)-\frac{x^{2} \beta^{2}}{2}\left[K_{1}(x)^{2}-K_{0}(x)^{2}\right]\right\} \tag{3}
\end{equation*}
$$

where $x=\frac{\xi b_{\text {min }}}{b}=\frac{E b_{\text {min }}}{\gamma \beta c \hbar}$. Notice in equation (3) that the contribution from the second pulse $P_{2}$ is multiplied by a factor of $\frac{1}{\gamma^{2}}$, which is hidden in the $\beta^{2}=1-\frac{1}{\gamma^{2}}$ coefficient. When $\beta \rightarrow 1$ as $v \rightarrow c$, the contribution from the second pulse vanishes.

The process we are interested in is the Coulomb fission cross section of the projectile ion $A$, which breaks up as it passes the target ion $B$ in the peripheral collision. This total cross section can be written using the notation of reference 10 as

$$
\begin{equation*}
\sigma_{A}^{C_{f}}=\int_{E_{\min }}^{E_{\max }} N_{B}(E) \sigma_{A}^{\gamma f}(E) d E \tag{4}
\end{equation*}
$$

where $\sigma_{A}^{\gamma f}(E)$ represents the microscopic photofission cross section, $C$ indicates Coulomb, $f$ fission, and $\gamma$ a single photon process. Equivalently, equation (1) can be substituted into equation (4) to find

$$
\begin{equation*}
\sigma_{A}^{C_{f}}=\int_{b_{\min }}^{\infty} 2 \pi b d b \int_{E_{\min }}^{E_{\max }} N_{B}(E, b) \sigma_{A}^{\gamma f}(E) d E \tag{5}
\end{equation*}
$$

which gives another form for the total Coulomb fission cross section.

## 3. The Photofission Cross Section

The photofission cross sections $\sigma_{A}^{\gamma f}(E)$ for $A=208$ lead and $A=209$ bismuth projectile ions were constructed according to reference 10 . Graphical functions for these photofission cross sections in the photon energy range from 20 MeV to 240 MeV were found in reference 15 . However, the overall semilog graphical functions for the energy range from 100 MeV to 1000 MeV were found in reference 16. For calculations, we took photofission points every 20 MeV in photon energy $E$. These functions are represented by the semilog point graphs in figure 1 for ${ }^{208} \mathrm{~Pb}$ and in figure 2 for ${ }^{209} \mathrm{Bi}$.

We extrapolated the photofission graphs by linear extensions of the semilog points to 2000 MeV according to reference 10 . The extrapolations are designed to capture the total
cross section results found in reference 10 when the approximate WW method of equation (9) is used. By reproducing their results we established photofission cross sections similar to those found in reference 10. To see how sensitive the total cross section calculation is to the particular extrapolation used we considered three cases with point indicators in comparison with reference 10: circle (high), square (similar), and diamond (low) as shown in figures 1 and 2 .

Figure 1 shows our points for the photofission cross section of ${ }^{208} \mathrm{~Pb}$ with three extrapolations beyond 1000 MeV . Figure 2 shows our points for the photofission cross section of ${ }^{209} \mathrm{Bi}$ with three extrapolations beyond 1000 MeV . The middle extrapolations with square or similar point indicators result in Coulomb fission cross sections that compare well with those reported in reference 10 .

## 4. Calculation of the Coulomb Fission Cross Section

The calculations of the Coulomb fission cross sections were done using a simple extended trapezoidal integration scheme. The energy variable $E$ was integrated from 20 MeV to 2000 MeV . The number of points used for the microscopic photofission cross section was 100 . The energy interval was 20 MeV to accommodate this range. The mimimum impact parameter for both collisions, $\mathrm{Pb}-\mathrm{Pb}$ and $\mathrm{Bi}-\mathrm{Pb}$, was chosen to be $b_{\text {min }}=15$ fm to correspond with the value used by reference 10 . Note the maximum photon energy occurs at $b_{\text {min }}$ and is given by $E_{\text {max }}=\frac{\gamma \hbar \beta c}{b_{\text {min }}}$.

We calculated cross sections using the full WW method by substituting equation (3) into equation (4) and integrating numerically as described with a standard modified Bessel function routine. We calculated cross sections using the approximate WW method by dropping the second term in equation (2). This is the contribution from the radiation pulse $P_{2}$ that is associated with the longitudinal electric field. This is equivalent to assuming $v=c$ or that $\beta=1$. The resulting expression contains the modified Bessel function $K_{1}(\xi)^{2}$. According to the asymptotic behavior of this function, the approximate expression can be split between a low photon energy and high photon energy expression. These energy approximations (ref. 10) are represented by

$$
\begin{equation*}
N_{B}(E, b) \simeq \frac{Z_{B}^{2} \alpha}{\pi^{2} \beta^{2} b^{2} E} \tag{6}
\end{equation*}
$$

for the low energy approximation $E \ll \frac{\gamma \hbar B c}{b}$ and

$$
\begin{equation*}
N_{B}(E, b) \simeq \frac{Z_{B}^{2} \alpha}{2 \pi \gamma \beta^{2} b} \exp \left(\frac{-2 E b}{\gamma \hbar \beta c}\right) \tag{7}
\end{equation*}
$$

for the high energy approximation $E \gg \frac{\gamma \hbar \beta c}{b}$.

Following reference 10 we used the low energy approximation of equation (6) and set $N_{B}(E, b)=0$ for energies $E \geq \frac{\gamma \hbar \beta c}{b}$. That is, we only integrate up to the cutoff photon energy. Substituting equation (6) into equation (5), this approximation to the full WW method gives the Coulomb fission cross section as

$$
\begin{equation*}
\sigma_{A}^{C_{f}}=\frac{2 Z_{B}^{2} \alpha}{\pi \beta^{2}} \int_{E_{\min }}^{E_{\max }} \sigma_{A}^{\gamma_{f}}(E) \frac{d E}{E} \int_{b_{\min }}^{\frac{\gamma h \beta c}{E}} \frac{d b}{b} \tag{8}
\end{equation*}
$$

Intergrating over the impact parameter $b$ results in

$$
\begin{equation*}
\sigma_{A}^{C_{f}}=\frac{2 Z_{B}^{2} \alpha}{\pi \beta^{2}} \int_{E_{\min }}^{E_{\max }} \sigma_{A}^{\gamma_{f}}(E) \ln \left(\frac{\gamma \hbar \beta c}{E b_{\min }}\right) \frac{d E}{E} \tag{9}
\end{equation*}
$$

Equation (9) was used to do the approximate WW numerical calculations corresponding to those of reference 10 .

Table 1 shows a comparison between our results and those of reference 10 for ${ }^{208} \mathrm{~Pb}$ and ${ }^{209} \mathrm{Bi}$ beam ions interacting with a ${ }^{208} \mathrm{~Pb}$ target. Column 1 gives the projectile ion; column 2 gives the results of reference 10 . Our approximate calculations using equation (9) are shown in column 5 . For ${ }^{208} \mathrm{~Pb}$ the calculation based on the similar type extrapolation of figure 1 gives 380 mb , which is comparable to the Coulomb fission cross section calculated by reference 10 . For ${ }^{209} \mathrm{Bi}$ the calculation based on the similar type extrapolation of figure 2 gives 450 mb , which is comparable to the value calculated in reference 10. Calculations using equation (9) based on the extrapolations represented by circle and diamond are, respectively, higher and lower, when compared with those of reference 10 . Convergence was checked by varying the number of integration points.

The experimental Coulomb fission cross section of 649 mb , shown in table 1, is our calculation based on reference 10. They calculated an expected yield of Coulomb fission events per incident ${ }^{208} \mathrm{~Pb}$ ion of $0.9 \times 10^{-2}$. The reference 10 calculation included the contributions of the isotopes ${ }^{208} \mathrm{~Pb},{ }^{207} \mathrm{~Pb}$, and ${ }^{206} \mathrm{~Pb}$, which were integrated through the $12-\mathrm{mm}$ thickness of the target. It was assumed that all the isotopes had the same Coulomb cross section of 380 mb . From the expected yield it is necessary to subtract off an 18-percent estimated correction for nuclear reinteraction of the fission fragments inside the target. However, the observed yield of Coulomb fission events per incident ${ }^{208} \mathrm{~Pb}$ ion was ( $1.26 \pm 0.16$ ) $\times 10^{-2}$ (from NA50 experiment at CERN SPS). Adjusting the observed yield number to before the 18 -percent correction for nuclear reinteraction would be about $1.536 \times 10^{-2}$, which is associated with an experimental cross section. This experimental Coulomb fission cross section becomes ( $\left.\frac{1.536}{0.9}\right) 380 \mathrm{mb} \approx 649 \mathrm{mb}$.

Table 1. Comparison of Coulomb Fission Cross Sections

| Ion <br> A | Ref. 10 <br> Calc., mb | Based <br> on Exp., mb | Figs. 1 \& 2 <br> $\sigma_{A}^{\gamma f}(E)$ | Approx. WW, mb <br> Eq. (9) | Full WW, mb <br> Eq. (3) in Eq. (4) |
| :--- | :---: | :---: | :--- | :---: | :---: |
| ${ }^{208} \mathrm{~Pb}$ |  |  | Circle | 395 | 331 |
| ${ }^{208} \mathrm{~Pb}$ | 380 | 649 | Square | 380 | 315 |
| ${ }^{208} \mathrm{~Pb}$ |  |  | Diamond | 361 | 296 |
| ${ }^{209} \mathrm{Bi}$ |  |  | Circle | 454 | 372 |
| ${ }^{209} \mathrm{Bi}$ | 450 |  | Square | 450 | 369 |
| ${ }^{209} \mathrm{Bi}$ |  |  | Diamond | 448 | 368 |

According to reference 10 the expected yield of Coulomb fission events per incident ${ }^{208} \mathrm{~Pb}$ ion after subtraction of an 18-percent correction for fission fragments is about $0.75 \times 10^{-2}$, which is about 40 percent lower than the observed yield. Based on our more accurate, full WW calculations for the Coulomb fission cross section for the ${ }^{208} \mathrm{~Pb}$ projectile of 315 mb shown in table 1, the revised calculated yield becomes $\left(\frac{315}{380}\right)(0.9 \times$ $\left.10^{-2}\right)(1.00-0.18) \approx 0.61 \times 10^{-2}$. This is now about 52 percent below the observed yield of $1.26 \times 10^{-2}$.

## 5. Calculation of the Electromagnetic Absorption Cross Section

The electromagnetic absorption (abs) cross section is calculated by replacing the photofission cross section $\sigma_{A}^{\gamma_{f}}(E)$ with the photoabsorption cross section $\sigma_{A}^{\gamma_{a b s}}(E)$ in the previous equations. We constructed a rough $\log -\log$ point graph of this absorption cross section for ${ }^{208} \mathrm{~Pb}$ from a graph found in reference 17 as shown in figure 3. Table 2 shows our results in comparison with reference 10 . Column 3 data compare well with that of column 2, which are also based on equation (9). The full WW method shown in column 4 results in a lower cross section.

Table 2. Electromagnetic Absorption Cross Section

| Ion <br> A | Ref. 10 <br> Calc., b | Approx. WW, b <br> Eq. (9) | Full WW, b <br> Eq. (3) in Eq. (4) |
| :---: | :---: | :---: | :---: |
| ${ }^{208} \mathrm{~Pb}$ | 50 | 49 | 44 |

Integration of equations (4) and (9) was done over five photon energy ranges due to the form of our log-log graph for the absorption cross section: from 6 MeV to 20 MeV with an energy increment of 1 MeV ; from 20 MeV to 200 MeV with an energy increment of 10 MeV ; from 200 MeV to 300 MeV with an energy increment of 20 MeV ; from 300 MeV to 1000 MeV with an energy increment of 100 MeV ; and from 1000 MeV to 2000 MeV with an energy increment of 200 MeV . We used the same trapezoidal integration scheme and convergence check as that discussed in section 4.

## 6. Discussion

The reason that the full WW method results in smaller Coulomb fission cross section compared with the approximate WW method is due to the slight difference in the respective WW number spectrums as shown in figure 4 . The $N(E)$ s match at low values and high values of $E$, but through the central energy range the full WW number spectrum is offset lower than the approximate WW number spectrum.

The calculated expected yield of Coulomb fission events per incident ${ }^{208} \mathrm{~Pb}$ projectile ion was reported to be 40 percent lower than the experimentally observed yield in reference 10. This was based upon their calculated Coulomb fission cross section of 380 mb . However, with our more accurate cross section of 315 mb , the discrepancy becomes worse: 52 percent lower for the calculated yield versus the observed yield. For an experimentally based effective cross section of about 649 mb , this implies that the physics of the $\mathrm{Pb}-\mathrm{Pb}$ interaction is still not understood.

The key to application of the WW method is the theoretical or experimentally based photonuclear cross section. Our photofission cross sections of figures 1 and 2 are reconstructions from inferred cross sections based on experimental electron-induced fission cross sections (ref. 16). After 1000 MeV we used linear semilog extrapolations up to 2000 MeV . Why the ${ }^{208} \mathrm{~Pb}$ graph is rising and the ${ }^{209} \mathrm{Bi}$ graph is decreasing at 1000 MeV would reflect the nature of the fission for these nuclei.

The choice of $b_{\text {min }}$ has an important effect on the resulting cross section. Smaller numbers for $b_{\text {min }}$ result in larger cross sections. In order to avoid inducing strong interactions $b_{\min }=R_{A}+R_{B}$. We used 15 fm for the minimum impact parameter, the value used in reference 10. We also looked at values of $b_{\min }$ based on other methods (refs. 4, 18, and 19). These methods resulted in about the same discrepancy as reported previously. Calculations using $b_{\text {min }}$ according to the Wood-Saxon method (refs. 20 and 21) resulted in larger cross sections, however, not large enough to remove the discrepancy.

The interference between nuclear and Coulomb forces is expected to be small because of the different distance behavior of the forces: the Coulomb force being weak and long range and the nuclear force being strong and short range. (This is discussed in ref. 18, where they show that the interference is small.) Thus, the question over the discrepancy between the calculated expected yield per Coulomb fission event and the experimentally observed yield remains open. However, because of previous excellent agreement with theory and experiment, we suspect that the solution of the disagreement lies with the complicated and approximate procedure used to extract the experimental cross sections.

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Figure 1. Photofission cross section for ${ }^{208} \mathrm{~Pb}$.


Figure 2. Photofission cross section for ${ }^{209} \mathrm{Bi}$.


Figure 3. Photoabsorption cross section for ${ }^{208} \mathrm{~Pb}$.


Figure 4. Comparison of the full (solid line) and approximate (dashed line) WW number spectrums per unit energy.



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