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## Nomenclature

Natural units used in this work are such that $\hbar = c = 1$.

| | |
|---|---|
| $c$ | speed of light |
| $cm$ | center of mass |
| $\frac{d\sigma}{dE}$ | spectral distribution cross section |
| $E$ | total energy of particle |
| $\frac{Ed^3\sigma}{dp^3}$ | Lorentz invariant differential cross section |
| $f(y)$ | transitional probability from one vertex to next with rapidity $y$ |
| $\hbar$ | Planck constant divided by $2\pi$ |
| $H(\Gamma)$ | probability distribution of vertex occurring at $\Gamma$ |
| $m$ | mass |
| $p$ | one-dimensional momentum |
| $p_{max}$ | maximum momentum of produced particles |
| $p_{\pm}$ | plus or minus light-cone momentum, $E \pm p$ |
| $s$ | Mandelstam variable representing total energy square in $cm$ frame, $(p_1 + p_2)^2$ or $\sqrt{s} = E_{1cm} + E_{2cm} = E_{cm}$ |
| $t$,$\tau$ | time |
| $V_n$ | $n$th spacetime position or "vertex" of breakup point along string, $\kappa(x_+, x_-)$ |
| $W_{\pm}$ | kinetic energy of quark or antiquark along $\pm$ light-cone coordinate |
| $x$ | coordinate of position or fragmentation variable, $\frac{p_z}{p_{max}}$ |
| $x_{\pm}$ | plus or minus light-cone configuration, $t \pm x$ |
| $y$ | rapidity, $\frac{1}{2}\ln\frac{E+p}{E-p}$ |
| $\gamma$ | Lorentz factor, $\frac{1}{\sqrt{1-v^2}}$ |
| $\Gamma$ | proper time, $\kappa x_+ x_- = \kappa(t^2 - x^2) = \kappa\tau^2$ |
| $\kappa$ | string constant |

## Abstract

*String fragmentation models such as the Lund Model fit experimental particle production cross sections very well in the high-energy limit. This paper gives an introduction of the massless relativistic string in the Lund Model and shows how it can be modified with a simple assumption to produce formulas for meson production cross sections for space radiation research. The results of the string model are compared with inclusive pion production data from proton-proton collision experiments.*

# 1. Introduction

The Lund model is a $(1+1)$ massless relativistic string fragmentation model which is modified with a simple assumption in this paper to produce formulas for meson particle production cross sections for HZETRN. The idea of modelling hadronic systems with strings went back to the 1960s (ref. 1). The general skepticism on the extra dimensions predicted by string theory and the advent of QCD in the 1970s put string theory out of commission for the next decade. Interests in string theory rekindled in the 1980s when Green and Shwarz showed that string theory is anomaly free and probably finite to all orders in perturbation theory. Today many advances in string theory have been made on the theoretical front. Despite all the excitement string theory has generated in the high-energy theory community, no evidence exists that string theory will make any low-energy limit prediction to be tested by experiments in the near future. Supersymmetry and superstring may be testable by experiments in the TeV scale but perhaps not directly relevant to the focus of space radiation research in the GeV scale. The interest lies mostly in the low-energy limit of a nonsupersymmetric string theory, with the possible exception of understanding the Greisen-Zatsepin-Kuzmin (GZK) cutoff in the ultrahigh-energy cosmic ray spectrum using TeV strings (ref. 2).

String fragmentation has been a work horse in analyzing high-energy particle production. Isgur proposed a fluxtube model in which the color force field is thought as a string-like object (refs. 3 and 4). B. Andersson implemented the idea of string fragmentation formally in the Lund Model (refs. 5, 6, and 7), which is implemented in the JETSET and PYTHIA Monte-Carlo programs (ref. 8). If a Monte-Carlo cross-section program is put into a target code, it will not run in a short time. Therefore we need simple parameterizations of cross-section formulas, which is the aim of the present work. In this paper, we first review the original Lund Model and expand some of the derivations of reference 5. Later we show how to modify the Lund Model by inserting a simple assumption to generate formulas for meson production cross sections.

Appendix A defines the basic kinematic notations. Appendix B explains the basic concepts of the Lund Model and derives the invariant amplitude formulas. The formulas in the appendixes are taken from reference 5, chapters 6 to 8. This work merely serves to focus on a subsection of reference 5 that is relevant to our discussion and expands the derivations. Section 2 of this paper explains a new idea on how to use a simple ansatz to obtain formulas for production cross sections in the Lund Model.

## 2. Cross-Section Formulas

The basic result of the Lund Model is the "Area Law" which is summarized as (ref. 5)

$$dP_{ext} = ds \frac{dz}{z} (1-z)^a e^{-b\Gamma} \tag{1}$$

$$dP_{int} = \prod_{j=1}^{n} N\, d^2p_{0j}\, \delta^+(p_{0j}^2 - m^2)\, \delta\left(p_{rest} - \sum_{j=1}^{n} p_{0j}\right) e^{-bA_{rest}} \tag{2}$$

and derived in appendix B. The symbols $\Gamma$ and $A_{rest}$ are Lorentz invariant kinematic variables. The variable $\Gamma$ defines the surface of constant proper time along which the string is broken. Traditionally the Lund Model results in equations (1) and (2) are incorporated into Monte Carlo simulation programs such as JETSET and PYTHIA to compute cross sections. Since the goal of the HZETRN program is to derive simple formulas for the cross sections, numerical calculations are not wanted. In this section, some simple assumptions are made to derive analytical results.

Equation (2) resembles the quantum mechanical result

$$d\sigma = d\Omega(s; p_{01}, \ldots, p_{0n})\, |\mathcal{M}|^2 \tag{3}$$

and (ref. 9)

$$d\sigma = \frac{(2\pi)^4}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}\, \delta^4\left(p_1 + p_2 - \sum_{i=3}^{n+2} p_i\right) \prod_{i=3}^{n+2} \frac{d^3p_i}{(2\pi)^3 2E_i}\, |\mathcal{M}|^2 \tag{4}$$

By analogy, it is assumed that (ref. 5)

$$|\mathcal{M}|^2 = e^{-bA_{rest}} \tag{5}$$

In principle, $A_{total} = \Gamma + A_{rest}$ can also be used in equation (5) in place of $A_{rest}$. The only difference is a proportionality constant $\exp(-b\Gamma)$ in the invariant amplitude $|\mathcal{M}|^2$. Hence the assumption in equation (5) is the simplest. The string breaks up along a surface of constant proper time $\Gamma$ which is determined by the distribution (ref. 5)

$$H(\Gamma) = C\,\Gamma^a\, e^{-b\Gamma}. \tag{6}$$

By setting $dH/d\Gamma = 0$, we can easily show that the maximum of $H(\Gamma)$ is (ref. 5)

$$\Gamma_{max} = \frac{a}{b} \tag{7}$$

The mean proper time is (ref. 5)

$$< \Gamma >= \frac{\int_0^\infty \Gamma\, H(\Gamma)\, d\Gamma}{\int_0^\infty H(\Gamma)\, d\Gamma} = \frac{a+1}{b} \tag{8}$$

The fact that $a$ and $b$ are constants in $\Gamma$ as shown in appendix B does not exclude the possibility that they are functions of rapidity. Since string fragmentation is a stochastic process that occurs over a surface of constant proper time on the average, $< \Gamma >$ is expected to be an absolute constant; this means that

$$a = a_0\, f(y) - 1 \tag{9}$$

$$b = b_0\, f(y) \tag{10}$$

for some function of rapidity $f(y)$. The goal of a consistent string theory is to calculate $a_0$, $b_0$, and $f(y)$ analytically. In this paper, the simple assumption is that

$$f(y) = \frac{2 m_T \sinh y}{\sqrt{s'}} = \frac{2 p_z}{\sqrt{s'}} \approx x, \tag{11}$$

where $m_T = \sqrt{m^2 + p_T^2}$. The kinematic identities of equation (11) are taken from reference 9. The ansatz $f(y) = x$ is motivated by a posteriori considerations when fitting experimental data. In NASA space radiation research, cosmic particles are highly energetic with production mostly forward. In the center-of-mass frame, the longitudinal momentum is $p_z$ and the total energy of the colliding particles is $\sqrt{s'}$. According to the Fermi Golden Rule,

$$\frac{d\sigma}{dx} \sim |\mathcal{M}|^2 \tag{12}$$

Together with equations (10) and (11), the differential cross section is found to be

$$\frac{d\sigma}{dx} = A\, e^{-Bx} \tag{13}$$

for some constants $A$ and $B$. This prediction is consistent with experimental data from references 10 and 11 as shown in figures 1 and 2.

The spectral distribution can be obtained from the cross-section formula in reference 12

$$\frac{d\sigma}{dE} = 2\pi p \int_0^{\theta_{max}} d\theta\, E \frac{d^3\sigma}{dp^3} \sin\theta \tag{14}$$

By using approximations $p_z \approx E$ and $x \sim p_z/\sqrt{s}$ in the high-energy limit and equation (12) in conjunction with equation (14), the spectral distribution is found to be

$$\begin{aligned} \frac{d\sigma}{dE} &\sim 2\pi(1 - \cos\theta_{max})\, p_z |\mathcal{M}|^2 \\ &= c\, E\, e^{-kE} \end{aligned} \tag{15}$$

where $c$ and $k$ are constants. Unfortunately no experimental data exist for $d\sigma/dE$ for pion production in proton-proton scattering. Blattnig et al. produced a parameterized spectral distribution by integrating $E\, d^3\sigma/dp^3$ (ref. 12).

## 3. Results

The Monte-Carlo programs JETSET and PYTHIA cannot be used in HZETRN due to excessive running time. Parametrizations of cross-section formulas are needed. The new idea is to make simple assumptions within the Lund Model framework to obtain analytic meson production cross-section formulas. Our purpose was not to produce any parameterization. To this end, all the fits in this work have parameters that are simply handpicked for the sake of illustrating the correctness of the qualitative aspects of cross sections predicted by string theory. Figures 1 and 2 clearly demonstrate the success of the fits. A major assumption in this paper is that $a$ and $b$ in equation (6) are not constants as proposed in the original Lund Model but are functions of rapidity. This assumption allows us to obtain the correct qualitative features of the cross sections without resorting to a Monte-Carlo simulation. At the present, the functions $a$ and $b$ are adjusted to fit the data.

The exact forms of the parameters $a$ and $b$ (eqs. (9) and (10)) for various types of production are calculated from nonsupersymmetric string theory, such as the QCD string model (ref. 13). The concept of confinement by a string is quantified in terms of the minimal area law of the Wilson loop. QCD string models also include the gluonic degrees of freedom. The constants $A$ and $B$ in equation (13) and the constants $c$ and $k$ in equation (15) are also calculated from string theory. The $(1+1)$ Lund Model cannot give angular dependence which is important for low $p_z$. Future work in string phenomenology hopefully can include the angular dependence of the cross sections near the threshold by extending the model to $(3+1)$ or higher dimensions. Production formulas will also be extracted from JETSET and PYTHIA to be used in HZETRN.

# Appendix A

## Kinematics

The quark-antiquark pair of a meson is massless in the string model. In this case, the concept of the mass of a meson is associated with the mass of the string field and not the quarks. Massless quarks move at the speed of light. This result is not surprising because string theory predicts that the ends of open strings move at the speed of light either by imposing a Neumann boundary condition for open strings (ref. 14) or by solving the classical equations of motion (ref. 15). In the highly relativistic problems, the light-cone coordinates are the natural choice. In the $(1+1)$ case, Lorentz transformations can be written as

$$\begin{pmatrix} E' \\ p' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix} \equiv \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix} \equiv \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix} \tag{17}$$

where $\gamma$ is the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-v^2}} \tag{18}$$

These equations imply

$$\gamma = \cosh y_p \tag{19}$$

$$\gamma v = \sinh y_p \tag{20}$$

$$v = \tanh y_p \tag{21}$$

The energy and momentum of a particle can now be written as

$$E = \gamma m = m \cosh y_p \tag{22}$$

$$p = \gamma m v = m \sinh y_p \tag{23}$$

The boosted energy $E_b$ and the boosted one-momentum $p_b$ are given as

$$\begin{aligned} E_b &= \gamma\,(E - v\,p) \\ &= m\,(\cosh y_p \cosh y - \sinh y_p \sinh y) \\ &= m\,\cosh(y_p - y) \end{aligned} \tag{24}$$

$$\begin{aligned} p_b &= \gamma\,(p - v\,E) \\ &= m\,(\sinh y_p \cosh y - \cosh y_p \sinh y) \\ &= m\,\sinh(y_p - y) \end{aligned} \tag{25}$$

The relativistic velocity addition formula is simple in light-cone coordinates (ref. 5) with

$$v' = \tanh y' = \frac{v - v_b}{1 - v_b v} = \tanh(y - y_b) \tag{26}$$

illustrating the additivity of rapidity,

$$y = y' + y_b \tag{27}$$

This simplication motivates the definition of rapidity

$$y_p = \frac{1}{2} \ln\left(\frac{1+v}{1-v}\right) = \frac{1}{2} \ln\left(\frac{E+p}{E-p}\right) \tag{28}$$

Momenta along the light-cone can be defined as (ref. 5)

$$p_+ = E + p = m \cosh y_p + m \sinh y_p = m\, e^{y_p} \tag{29}$$

$$p_- = E - p = m \cosh y_p - m \sinh y_p = m\, e^{-y_p} \tag{30}$$

With these definitions, boosts are simplified along the light cone (ref. 5),

$$p'_\pm = m\, e^{\pm(y_p - y)} = p_\pm e^{\mp y} \tag{31}$$

Light cone coordinates can also be defined in configuration space as

$$t \equiv \frac{m}{\kappa} \cosh y_q \tag{32}$$

$$x \equiv \frac{m}{\kappa} \sinh y_q \tag{33}$$

where $\kappa$ is the string constant and is used here to give $t$ and $x$ the correct dimension. A new subscript is adopted for $y_q$ in configuration space to distinguish $y_p$ in momentum space. These definitions are consistent with the requirement that $v = dx/dt = \tanh y_q$, as in equation (21). Similar results are obtained as in the momentum space case,

$$x_+ = t + x = \frac{m}{\kappa} \cosh y_q + \frac{m}{\kappa} \sinh y_q = \frac{m}{\kappa}\, e^{y_q} \tag{34}$$

$$x_- = t - x = \frac{m}{\kappa} \cosh y_q - \frac{m}{\kappa} \sinh y_q = \frac{m}{\kappa}\, e^{-y_q} \tag{35}$$

$$x'_\pm = \frac{m}{\kappa}\, e^{\pm(y_q - y)} = x_\pm e^{\mp y} \tag{36}$$

## Appendix B

## Lund Model

The focus of this paper is primarily mesons. The force field between a quark-antiquark pair is presumed to be constant and confined to a flux-tube. This appendix is based on the work of reference 5, chapters 6 to 8. The equation of motion of any one member of the quark-antiquark pair acted upon by a constant force is (ref. 5)

$$F = \frac{dp}{dt} = -\kappa \tag{37}$$

where $\kappa$ is the string constant. The solution of the equation is

$$p(t) = p_0 - \kappa t = \kappa(t_0 - t) \tag{38}$$

From $E^2 = p^2 + m^2$, $E\,dE = p\,dp$ or

$$\frac{p}{E} = \frac{dE}{dp} \tag{39}$$

is obtained. There is also

$$p = \gamma m v = \gamma m \frac{dx}{dt} = E \frac{dx}{dt} \tag{40}$$

Equations (39) and (40) together yield

$$\frac{dx}{dt} = \frac{p}{E} = \frac{dE}{dp} \tag{41}$$

such that

$$\frac{dE}{dx} = \frac{dE}{dp}\frac{dp}{dt}\frac{dt}{dx} = \frac{dp}{dt} = -\kappa \tag{42}$$

Its solution gives the expected QCD flux tube result

$$E = \kappa(x_0 - x) \tag{43}$$

Combining equations (38) and (43) and $m = 0$, the relativistic equation of state is

$$m^2 = E^2 - p^2 = \kappa[(x_0 - x)^2 - (t_0 - t)^2] = 0 \tag{44}$$

Let $x_0 = t_0 = 0$, the equation of motion $|x| = t$ is obtained. The maximum kinetic energy at $x = 0$ puts an upper bound on the displacement of the particle such that $\kappa x_{max} = W$. The path of the particle is illustrated in figure 3. The zig-zag motion of the particle is the reason for the name "yoyo state." The quark-antiquark pair of a meson are assumed to be massless. The mass square of the meson is proportional to the area of the rectangle defined by the trajectories of the quark-antiquark pair. The period $\tau$ of the yoyo motion is

$$\tau = \frac{2E_0}{\kappa} \tag{45}$$

where $E_0$ is the maximum kinetic energy of each quark. In a boosted Lorentz frame, the period is transformed as

$$\tau' = \tau \cosh y \tag{46}$$

The breakup of a string occurs along a curve of constant proper time such that the process is Lorentz invariant. The Lund model assumes that the produced mesons are ranked, meaning that the production of the $n$th rank meson depends on the existence of rank $n-1$, $n-2$, ... , 1 mesons. A vertex $V$ is a breakup point and the location $\kappa(x_+, x_-)$ where a quark-antiquark pair is produced. The breakup of a string is represented in figure 4. The produced particle closer to the edges are the faster moving ones corresponding to larger rapidities.

Let $p_{\pm 0}$ and $p_{\pm j}$ be momentum of the parent quark and the $j$th of the $n$-produced quarks moving along the $x_\pm$ light-cone coordinate, respectively. Then

$$p_{\pm 0} = \sum_{j=1}^{n} p_{\pm j} \tag{47}$$

and the momentum fraction at $V_j$ is defined as

$$z_{\pm j} = \frac{p_{\pm j}}{p_{\pm 0}} \tag{48}$$

In order to simplify notations, $z_j$ equals $z_{+j}$ unless specified otherwise. The invariant interval is

$$ds^2 = dt^2 - dx^2 = dt^2(1 - v^2) \tag{49}$$

giving

$$ds = dt\sqrt{1 - v^2} = \frac{dt}{\gamma} = d\tau \tag{50}$$

where $\tau$ is the proper time. The proper time is also $x_+ x_- = (t - x)(t + x) = t^2 - x^2 = \tau^2$ and can be used as a dynamic variable such that

$$\Gamma = \kappa^2 x_+ x_- \tag{51}$$

Since $\Gamma$ is proportional to the proper time square $\tau^2$, its Lorentz invariant property makes it a good kinematic variable in the light-cone coordinates. Let $W_{\pm 1}$ and $W_{\pm 2}$ be the kinetic energies along $x_\pm$ at vertices 1 and 2, respectively. The following identities can be easily proven geometrically by calculating the areas of the rectangles $\Gamma_1 = A_1 + A_3$, $\Gamma_2 = A_2 + A_3$, and $m^2$ as shown in figure 5:

$$\begin{aligned}\Gamma_1 &= (1 - z_-)W_{-2}W_{+1} & (52)\\ \Gamma_2 &= (1 - z_+)W_{-2}W_{+1} & (53)\\ m^2 &= z_- z_+ W_{-2}W_{+1} & (54)\end{aligned}$$

where $m$ is the mass of the produced meson. Equations (52) and (53) can be expressed with the help of equation (54) as

$$\begin{aligned}\Gamma_1 &= \frac{m^2(1 - z_-)}{z_+ z_-} & (55)\\ \Gamma_2 &= \frac{m^2(1 - z_+)}{z_+ z_-} & (56)\end{aligned}$$

Differentiating these equations gives

$$\begin{aligned}\frac{\partial \Gamma_1}{\partial z_-} &= -\frac{m^2}{z_+ z_-^2} & (57)\\ \frac{\partial \Gamma_2}{\partial z_+} &= -\frac{m^2}{z_+^2 z_-} & (58)\end{aligned}$$

Let $H(\Gamma_1)$ be a probability distribution such that $H(\Gamma_1)\, d\Gamma_1\, dy_1$ is the probability of a quark-antiquark pair being produced at the spacetime position $V_1$. From now on, the symbol $V_n$ also represents the breakup event that leads to the creation of the $n$th quark-antiquark pair along a surface of constant $\tau$. Let $f(z_+)dz_+$ be the transition probability of obtaining $V_2$ given that $V_1$ occurs. The transition probability of $V_2$ via $V_1$ is then $H(\Gamma_1)\, d\Gamma_1\, dy_1\, f(z_+)dz_+$. Similarly the probability of $V_1$ via $V_2$ is $H(\Gamma_2)\, d\Gamma_2\, dy_2\, f(z_-)dz_-$. It is reasonable to assume that the probability of $V_1$ via $V_2$ is equal to that of $V_2$ via $V_1$ such that

$$H(\Gamma_1)\, d\Gamma_1\, dy_1\, f(z_+)dz_+ = H(\Gamma_2)\, d\Gamma_2\, dy_2\, f(z_-)dz_- \quad (59)$$

The Jacobian $J$ in

$$d\Gamma_1\, dz_+ = J\, d\Gamma_2\, dz_- \quad (60)$$

is

$$J = \left\| \begin{matrix} \frac{\partial \Gamma_1}{\partial \Gamma_2} & \frac{\partial \Gamma_1}{\partial z_-} \\ \frac{\partial z_+}{\partial \Gamma_2} & \frac{\partial z_+}{\partial z_-} \end{matrix} \right\| = \frac{z_+}{z_-} \quad (61)$$

which can be easily computed by using equations (57) and (58) and $\partial\Gamma_1/\partial\Gamma_2 = \partial z_+/\partial z_- = 0$. Equation (60) can now be reexpressed as

$$d\Gamma_1 \frac{dz_+}{z_+} = d\Gamma_2 \frac{dz_-}{z_-} \quad (62)$$

Since rapidity is additive ( i.e., $y_2 = y_1 + \Delta y$)

$$dy_1 = dy_2 \tag{63}$$

With equations (62) and (63), equation (59) is simplified as

$$H(\Gamma_1) z_+ f(z_+) = H(\Gamma_2) z_- f(z_-) \tag{64}$$

New definitions are now made to facilitate the solution of the equation

$$h(\Gamma) = \log H(\Gamma) \tag{65}$$
$$g(z) = \log(z f(z)) \tag{66}$$

The new definitions transform equation (64) as

$$h(\Gamma_1) + g(z_+) = h(\Gamma_2) + g(z_-) \tag{67}$$

Notice that

$$\frac{\partial^2 g(z_+)}{\partial z_+ \partial z_-} = \frac{\partial^2 g(z_-)}{\partial z_+ \partial z_-} = 0 \tag{68}$$

Differentiate equation (67) with respect to $z_+$ and $z_-$, eliminate terms with $\partial^2 g/\partial z_+ \partial z_-$ using equation (68) and cancel out factors of $\partial\Gamma/\partial z$ on both sides of the equation to obtain

$$\frac{\partial h(\Gamma_1)}{\partial \Gamma_1} + \Gamma_1 \frac{\partial^2 h(\Gamma_1)}{\partial \Gamma_1^2} = \frac{\partial h(\Gamma_2)}{\partial \Gamma_2} + \Gamma_2 \frac{\partial^2 h(\Gamma_2)}{\partial \Gamma_2^2} \tag{69}$$

or equivalently

$$\frac{d}{d\Gamma}\left(\Gamma \frac{dh}{d\Gamma}\right) = -b \tag{70}$$

where $b$ is a constant. The solution is

$$h(\Gamma) = -b\Gamma + a \ln \Gamma + \ln C \tag{71}$$

where $C$ is a constant of integration. It yields the distribution

$$H(\Gamma) = e^{h(\Gamma)} = C\,\Gamma^a\, e^{-b\Gamma} \tag{72}$$

Substituting equations (55) and (56) into equation (67) gives

$$\begin{aligned} g_{12}(z_+) + \frac{bm^2}{z_+} + a_1 \ln \frac{m^2}{z_+} - a_2 \ln \frac{1 - z_+}{z_+} + \ln C_1 \\ = g_{21}(z_-) + \frac{bm^2}{z_-} + a_2 \ln \frac{m^2}{z_-} - a_1 \ln \frac{1 - z_-}{z_-} + \ln C_2 \end{aligned} \tag{73}$$

where $g_{12}(z_+)$ is $g(z_+)$ of $V_1$ changing to $V_2$ and $g_{21}(z_-)$ is $g(z_-)$ of $V_2$ changing to $V_1$. If $a = a_1 = a_2$, the transition probability distribution is

$$f(z_j) = N\,\frac{1}{z_j}\,(1-z_j)^a\,e^{-\frac{bm^2}{z_j}} \tag{74}$$

where $N$ is a constant of integration. Otherwise, the transition probability from $V_\alpha$ to $V_\beta$ is

$$f_{\alpha\beta}(z_j) = N_{\alpha\beta}\,\frac{1}{z_j}\,z_j^{a_\alpha}\,\left(\frac{1-z_j}{z_j}\right)^{a_\beta}\,e^{-\frac{bm^2}{z_j}} \tag{75}$$

where $N_{\alpha\beta}$ is a constant of integration specific to the vertices $V_\alpha$ and $V_\beta$. Let $z_{0j}$ be the $j$th rank momentum fraction scaled with respect to $p_{+0}$, then

$$z_{01} = z_1 \tag{76}$$

$$z_{02} = z_2(1-z_1) \tag{77}$$

The probability of two dependent events is the product of the probabilities of the two individual events. The existence of the rank-2 hadron depends on that of the rank-1 hadron according to the Lund Model so that joint probability of their mutual existence is the product of the two individual probabilities of $V_1$ and $V_2$. With equations (75) to (77) the combined distribution of the rank-1 and -2 hadrons is

$$\begin{aligned} f(z_1)dz_1\,f(z_2)dz_2 &= f(z_{01})dz_{01}\,f\left(\frac{z_{02}}{1-z_{01}}\right)\frac{dz_{02}}{1-z_{01}} \\ &= \frac{N\,dz_{01}}{z_{01}}\,\frac{N\,dz_{02}}{z_{02}}(1-z_{01})^a\left(1-\frac{z_{02}}{1-z_{01}}\right)^a\,e^{-\frac{bm^2}{z_1}-\frac{bm^2}{z_2}} \\ &= \frac{N\,dz_{01}}{z_{01}}\,\frac{N\,dz_{02}}{z_{02}}(1-z_{01}-z_{02})^a\,e^{-b(A_1+A_2)} \end{aligned} \tag{78}$$

The geometrical identity $A_j = bm^2/z_j$ is used in the last step. Generalizing the product of two vertices to that of $n$ vertices, the differential probability for the production of $n$ particles is easily seen as

$$dP(1,\ldots,n) = (1-z)^a\prod_{j=1}^{n}\frac{N\,dz_{0j}}{z_{0j}}\,e^{-bA_j} \tag{79}$$

where $z = \sum_{j=1}^{n} z_{0j}$; let $p_{0j} = z_{0j}\,p_{+0}$ and $d^2p = dp_+\,dp_-$, and use the identity

$$\int dC\,dB\,\delta(BC-D) = \frac{dB}{B} \tag{80}$$

to obtain

$$\frac{dz_{01}}{z_{01}}\,\frac{dz_{02}}{z_{02}} = d^2p_{01}\,d^2p_{02}\delta^+(p_{01}^2-m^2)\,\delta^+(p_{01}^2-m^2) \tag{81}$$

With equation (81), equation (79) can be rewritten as (ref. 5)

$$dP(p_{01},\ldots,p_{0n}) = (1-z)^a \prod_{j=1}^{n} N\, d^2p_{0j}\, \delta^+(p_{0j}^2 - m^2)\, e^{-bA_j} \tag{82}$$

From simple geometry in figure 6, the kinetic energies of the quarks along the $\pm$ light cones are shown to be

$$W_+ = z\,p_{0+} \tag{83}$$

$$W_- = \sum_{j=1}^{n} \frac{m_j^2}{z_{0j}p_{0+}} \tag{84}$$

and the total kinetic energy square at $V_n$ is

$$s = W_+W_- = \sum_{j=1}^{n} \frac{m^2 z}{z_{0j}} \tag{85}$$

The total differential probability of $n$-particle production is (ref. 5)

$$\begin{aligned} & dP(z,s;p_{01},\ldots,p_{0n}) \\ = & \; dz\,\delta\left(z - \sum_{j=1}^{n} z_{0j}\right) ds\,\delta\left(s - \sum_{j=1}^{n}\frac{m^2 z}{z_{0j}}\right) dP(p_{01},\ldots,p_{0n}) \\ = & \; \frac{dz}{z}\,\delta\left(1 - \sum_{j=1}^{n} u_j\right) ds\,\delta\left(s - \sum_{j=1}^{n}\frac{m^2}{u_j}\right) dP(p_{01},\ldots,p_{0n}) \end{aligned} \tag{86}$$

where $u_j \equiv p_{0+j}/W_{n+}$. Use the method

$$\begin{aligned} & \delta\left(1 - \sum_{j=1}^{n}\frac{z_{0j}}{z}\right)\,\delta\left(s - \sum_{j=1}^{n}\frac{m^2}{u_j}\right) \\ = & \; \delta\left(W_{n+} - W_{n+}\sum_{j=1}^{n} u_j\right)\,\delta\left(W_{n-} - \sum_{j=1}^{n}\frac{m^2}{u_jW_{n+}}\right) \\ = & \; \delta\left(W_{n+} - \sum_{j=1}^{n} p_{0j+}\right)\,\delta\left(W_{n-} - \sum_{j=1}^{n} p_{0j-}\right) \\ \equiv & \; \delta^2\left(p_{rest} - \sum_{j=1}^{n} p_{0j}\right) \end{aligned} \tag{87}$$

where $p_{rest} = (W_{n+}, W_{n-})$, to reorganize equation (86) as

$$dP(z, s; p_{01}, \ldots, p_{0n}) = ds \frac{dz}{z} (1-z)^a e^{-b\Gamma} \delta \left( p_{rest} - \sum_{j=1}^{n} p_{0j} \right) \times \prod_{j=1}^{n} N \, d^2 p_{0j} \, \delta^+(p_{0j}^2 - m^2) \, e^{-bA_{rest}} \tag{88}$$

with the definition

$$A_{total} \equiv \sum_{j=1}^{n} A_j = \Gamma + A_{rest} \tag{89}$$

The claim is that $A_{total}$ is Lorentz invariant and is called the "Area Law." (See ref. 7.) Equation (88) can be separated in the external and internal parts as in reference 5:

$$dP_{ext} = ds \frac{dz}{z} (1-z)^a e^{-b\Gamma}, \tag{90}$$

$$dP_{int} = \prod_{j=1}^{n} N \, d^2 p_{0j} \, \delta^+(p_{0j}^2 - m^2) \, \delta \left( p_{rest} - \sum_{j=1}^{n} p_{0j} \right) e^{-bA_{rest}} \tag{91}$$

The external part contains kinematic variables $s$, $z$, and $\Gamma$. The internal part contains dynamic variables $p_{0j}$. Equations (90) and (91) are the final results.

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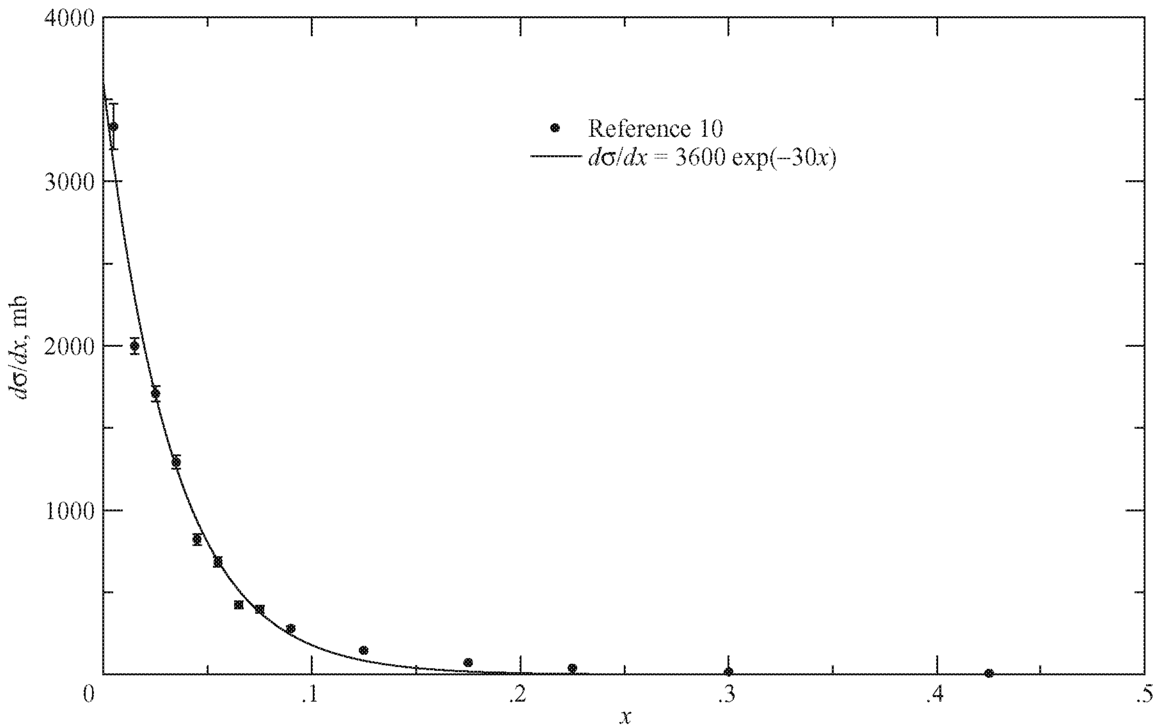


Figure 1. Experimental $d\sigma/dx$ and string model results. Constants used in exponential function chosen to fit data and not parameterizations; $p + p \rightarrow \pi^{\circ} + X$; $P_{\text{lab}}$ = 360 GeV.

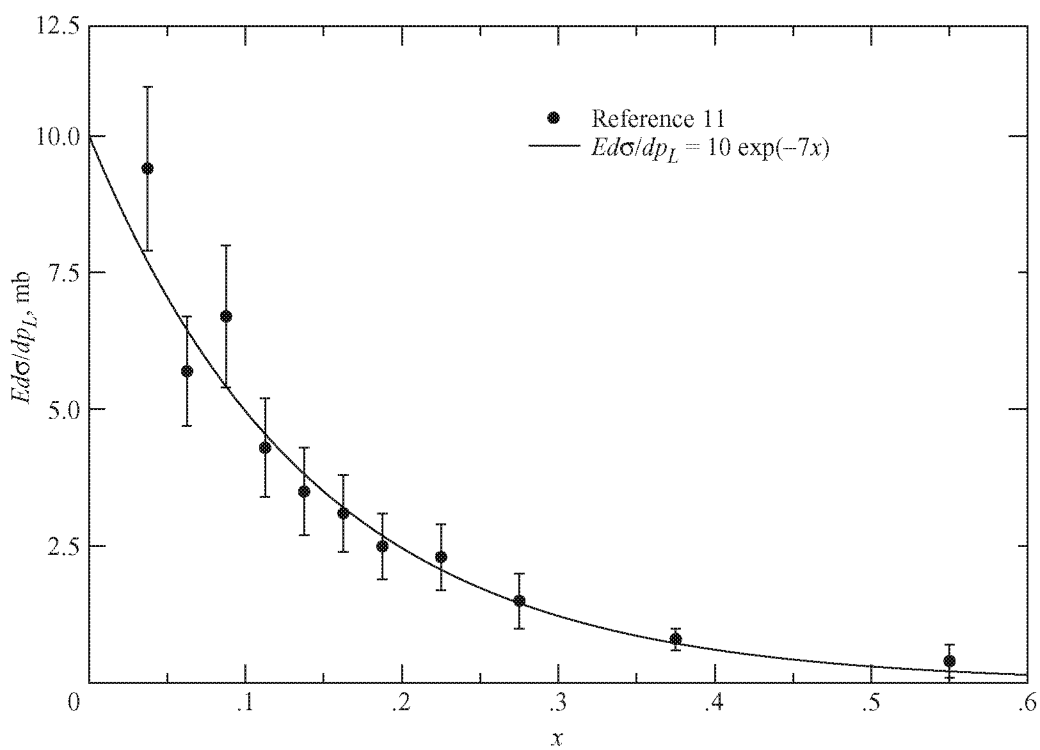


Figure 2. Experimental $E\, d\sigma/dp_L$ and string model results. Constants used in exponential function chosen to fit data and not parameterizations; $p + p \rightarrow \pi^{\circ} + X$; $P_{\text{lab}}$ = 250 GeV.

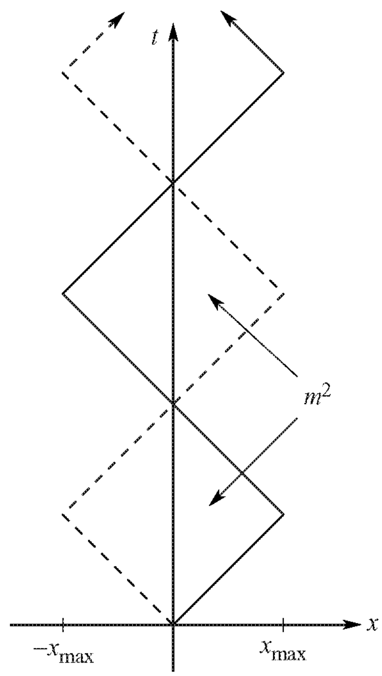


Figure 3. Yoyo motion of quark-antiquark pair confined by linear potential.

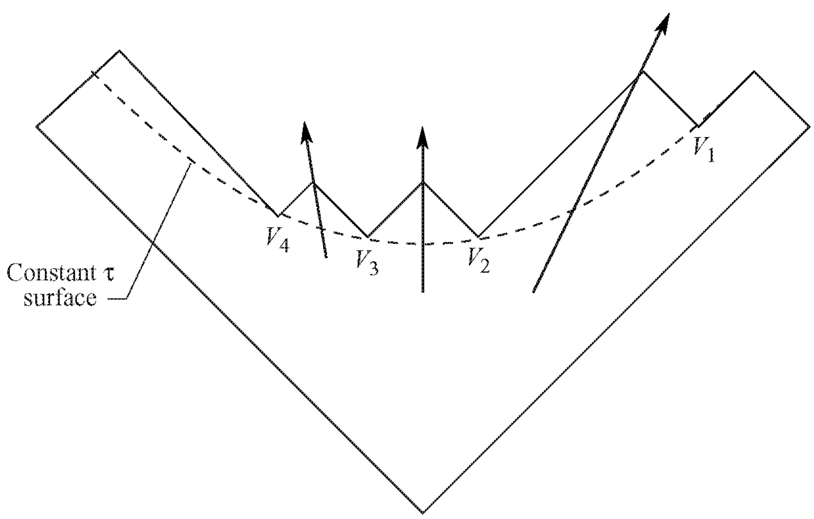


Figure 4. Breakup of quark-antiquark pair along surface of constant proper time $\tau$. Bold arrows represent velocities of produced mesons; breakup points labeled as vertices $V_1$ to $V_n$.

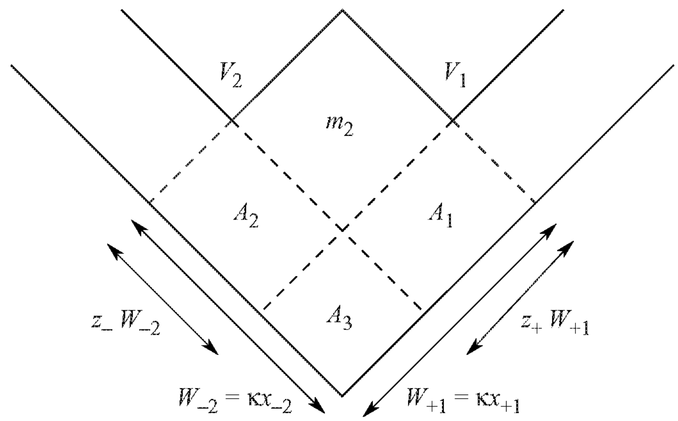


Figure 5. Geometry of kinematics of two adjacent vertices $V_1$ and $V_2$, which are vertices or spacetime positions that also represent energy carried by string field. Quark moves along positive light cone and antiquark moves along negative light cone; antiquark of $V_1$ combines with quark of $V_2$ to produce meson of mass $m$; $W_{+1}$ is energy of $V_1$ along $x_+$; $z_+W_{+1}$ is fraction of energy used to create quark from $V_2$; $W_{-2}$ is energy of $V_2$ along $x_-$; $z_-W_{-2}$ is fraction of energy used to create antiquark from $V_1$; $A_1, A_2, A_3$, and $m^2$ are areas of rectangles. From reference 5.

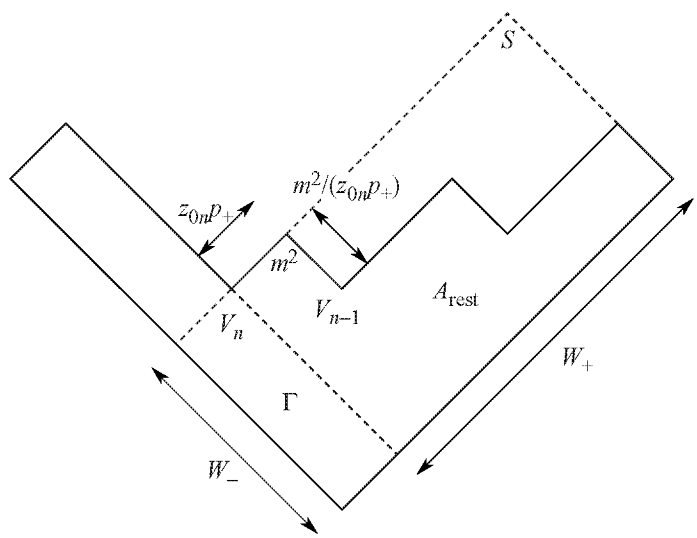


Figure 6. Geometry of kinematics of involving total area of spacetime diagram such that $A_{total} = \Gamma + A_{rest}$. Total energy square $s = W_+W_-$ is represented by rectangle $s$; $p\pm$ is energy of parent quark (antiquark) along $\pm$ light-cone coordinates; $z_{0n}p_+$ is fraction of energy used to create quark from $V_n$; $m^2/(z_{0n}p_+)$ is energy used to create antiquark from $V_{n-1}$. From reference 5 with minor modifications.

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**14. ABSTRACT**
String fragmentation models such as the Lund Model fit experimental particle production cross sections very well in the high-energy limit. This paper gives an introduction of the massless relativistic string in the Lund Model and shows how it can be modified with a simple assumption to produce formulas for meson production cross sections for space radiation research. The results of the string model are compared with inclusive pion production data from proton-proton collision experiments.

**15. SUBJECT TERMS**
String fragmentation model; High energy limit; Lund model; Meson production cross section in proton-proton collisions

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