The Astrophysical Journal Supplement Series, 182:120–126, 2009 May doi:10.1088/0067-0049/182/1/120

PARAMETERIZATIONS OF INCLUSIVE CROSS SECTIONS FOR KAON, PROTON, AND ANTIPROTON PRODUCTION IN PROTON–PROTON COLLISIONS

John W. Norbury
NASA Langley Research Center, Hampton, VA 23681, USA; john.w.norbury@nasa.gov
Received 2009 January 28; accepted 2009 February 25; published 2009 April 15

ABSTRACT

Inclusive kaon, proton, and antiproton production from high-energy proton–proton collisions is studied. Various available parameterizations of Lorentz-invariant, differential cross sections, as a function of transverse momentum and rapidity, are compared with experimental data. This paper shows that the Badhwar parameterization provides the best fit for charged kaon production. For proton production, the Alper parameterization is best and for antiproton production the Carey parameterization works best. The formulae for these cross sections are suitable for use in high-energy cosmic ray transport codes.

Key words: elementary particles – methods: analytical – methods: data analysis – methods: miscellaneous – radiation mechanisms: general

Online-only material: color figures

1. INTRODUCTION

Parameterizations of hadron production in high-energy proton–proton collisions find a variety of uses in astrophysics (Kaufman Bernado 2005; Domingo-Santamaria & Torres 2005; Kelner et al. 2006; Kamae et al. 2006; Moskalenko 2004; Prodanovic et al. 2007), nuclear physics (Blume 2007; d'Enterria 2005), simulations of particle physics experiments (Fasso et al. 2003; Allison et al. 2006), and space radiation applications (Wilson et al. 1991). Many π^0 mesons, which decay into two photons, arise from the decay of the produced hadrons. These photons can be detected with space-based γ-ray telescopes, and the hadron production cross sections enable one to calculate the spectrum of photons. Cross-section parameterizations have been used to calculate the γ-ray emission from the accretion disk around a black hole (Mahadevan et al. 1997), the diffuse γ-ray background, and the γ-ray spectrum from microquasars (Kaufman Bernado 2005). Recent works (Erlykin 2007; Giller 2008) emphasize the importance of accurate hadron production models for simulating extensive air showers. Such calculations generally involve Monte Carlo transport models, which use cross-section parameterizations as input. Accurate simulations of the air shower help to determine the primary cosmic ray spectrum, which helps in deducing the sources of high-energy cosmic rays (Erlykin 2007; Giller 2008). Space radiation protection is also another area where hadron production cross sections may find increasing importance (Wilson et al. 1991), especially for astronaut protection on the Martian surface, where particles reaching the ground have been transported through the Martian atmosphere. Accurate estimates of the surface radiation environment, require use of hadron production cross sections in radiation transport codes (Wilson et al. 1995).

Pion production parameterizations in high-energy proton–proton collisions have recently been studied (Blattnig et al. 2000; Norbury & Townsend 2007). These are presented in arithmetic form, which makes them convenient to use in transport codes. Huang et al. (2007) have recently emphasized the need for inclusion of all hadron production mechanisms in proton–proton collisions. Their application involves determining individual source contributions to the diffuse γ-ray background. Including the full set of hadrons, also enables more accurate calculations in the applications referred to previously. This is especially true well above the pion threshold, which is the case for most cosmic ray interactions and nuclear and particle physics experiment simulations.

In the present work, the pion parameterizations studied previously (Blattnig et al. 2000; Norbury & Townsend 2007) are extended to include the production of kaons, protons, and antiprotons. The formulae developed by Badhwar et al. (1977), Alper et al. (1975), Ellis & Stroynowski (1977), and Carey et al. (1974) will be compared to experimental data (Alper et al. 1975), at the energies listed in Table 1. These parameterizations, and the particles for which cross sections are available, are summarized in Table 2. The notation for the particles is as follows. Positive and negative charged kaons will be denoted as K^+ and K^-, respectively. Protons and antiprotons will be denoted as p^+ and p^-.

2. PARAMETERIZATIONS

The various available parameterizations for kaon, proton, and antiproton production in high-energy proton–proton collisions will now be reviewed. Some of the formulae have been listed previously (Blattnig et al. 2000; Norbury & Townsend 2007), but they will be repeated here for completeness. The paper by Norbury & Townsend (2007) also contains an extensive treatment of the kinematic variables introduced below.

2.1. Badhwar Parameterization

This parameterization (Badhwar et al. 1977) gives the Lorentz-invariant differential cross section for $K^\pm$ production as

$$E\frac{d^3\sigma}{d^3p}(K^\pm) = A(1-\tilde{x})^C \exp(-Bp_T), \quad (1)$$

where E is the energy, σ is the cross section, p is the momentum, and p_T is the transverse momentum. A, B, C are constants listed in Table 3. The $\tilde{x}$ variable is

$$\tilde{x} \equiv \left[x_F^2 + \frac{4}{s}\left(p_T^2 + m^2\right)\right]^{1/2}, \quad (2)$$

Table 1
Energies for Proton–Proton Collisions

$\sqrt{s}$	T_{lab}	p_{lab}
17	157	158
23	280	281
31	510	511
45	1077	1078
53	1495	1496
63	2113	2114

Notes. $\sqrt{s}$ is the total center of momentum energy and T_{lab} and p_{lab} are the proton projectile kinetic energy and momentum as measured in the target (lab) frame. All quantities are in units of GeV.

Table 2
Available Parameterizations

Particle	Badhwar	Alper	Ellis	Carey
K^+	Yes	Yes	Yes	No
K^-	Yes	Yes	Yes	Yes
p^+	No	Yes	Yes	No
p^-	No	Yes	Yes	Yes

Table 3
Constants for Badhwar Equation (1)

Particle	A	B	C
K^+	8.85	4.05	2.5
K^-	9.3	3.8	8.3

where x_F is the Feynman scaling variable (Norbury & Townsend 2007), s is the Mandelstam variable giving the square of the center of momentum energy, and m is the kaon mass.

2.2. *Alper Parameterization*

The Alper et al. (1975) parameterization for $K^\pm$ and $p^\pm$ production is

$$E\frac{d^3\sigma}{d^3p} = A_1 \exp(-Bp_T)\exp(-Dy^2) + A_2\frac{(1-p_T/p_{\mathrm{beam}})^m}{\left(p_T^2+M^2\right)^n}, \quad (3)$$

where y is the rapidity, and p_{beam} is beam momentum. Other quantities are listed in Table 4.

2.3. *Ellis Parameterization*

The Ellis & Stroynowski (1977) parameterization for $K^\pm$ and $p^\pm$ production is

$$E\frac{d^3\sigma}{d^3p} = A\left(p_T^2+M^2\right)^{-N/2}(1-x_T)^F, \quad (4)$$

where A is an overall normalization fitted to be $A = 13$ by Blattnig et al. (2000) and $x_T \equiv p_T/p_{\max} \approx 2p_T/\sqrt{s}$, with $p_{\max}$ being the maximum value of transferred momentum (Norbury & Townsend 2007). The same value of A is used in the present work. The other constants are listed in Table 5.

Table 4
Constants for Alper Equation (3)

Particle	A_1	B	D	A_2	M	m	n
K^+	14.3	6.78	1.5	8.0	1.29	12.1	4.0
K^-	13.4	6.51	1.8	9.8	1.39	17.4	4.0
p^+	5.3	3.8	−0.2	16	1.2	0	7.5
p^-	1.89	4.1	2.3	25	1.41	25	4.5

Table 5
Constants for Ellis Equation (4)

Particle	N	M^2	F
K^+	8.72	1.69	9.0
K^-	8.76	1.77	12.2
p^+	10.38	1.82	7.3
p^-	9.10	1.17	14.0

Table 6
Constants for Carey Equation (5)

Particle	N	h	G	J
K^-	13	0.36	1.22	5
p^-	13	0.26	1.04	7

2.4. *Carey Parameterization*

The Carey et al. (1974) parameterization, for K^- and p^- production is

$$E\frac{d^3\sigma}{d^3p} = hN\left(p_T^2+G\right)^{-4.5}(1-x_R)^J, \quad (5)$$

where N is an overall normalization fitted to be $N = 13$ by Blattnig et al. (2000) and $x_R \equiv p/p_{\max} \approx 2p/\sqrt{s}$. The same value of N is used in the present work. The constants are listed in Table 6.

3. COMPARISON TO EXPERIMENT

The various parameterizations have all been compared to the experimental results of Alper et al. (1975) at the energies listed in Table 1. Some of the comparisons are shown in Figures 1–18. If all the parameterizations were compared to all the data, there would be over 50 figures to look at. Of course, all of these figures have been analyzed, but they are not all presented herein for the sake of brevity. Note the following. For $K^\pm$ production, the comparisons to experiment for K^+ mesons are of very similar quality to those of the K^- mesons. Therefore, only results for K^- production are shown, because the Carey parameterization is not available for K^+ mesons. The results for p^+ and p^- production are quite different, so both will be shown. Five energies are available, namely $\sqrt{s} = 23, 31, 45, 53$, and 63 GeV. However, the results for 23 and 31 GeV are of very similar quality. It is preferable to show the lowest energy results, but the 31 GeV results are shown because more data are available at high p_T. Also the results for 45, 53, and 63 GeV are again of similar quality. It is preferable to show the highest energy results, but only the 53 GeV results are shown, because the data at 63 GeV do not contain high p_T data points. Now consider how well the various parameterizations agree with experiment.

Kaon results, versus experiment, are shown in Figures 1–8, for inclusive K^- production in proton–proton collisions at various energies. The rapidity for the top curve is $y = 0.0$, and it increases in steps of 0.2 from the top to the bottom

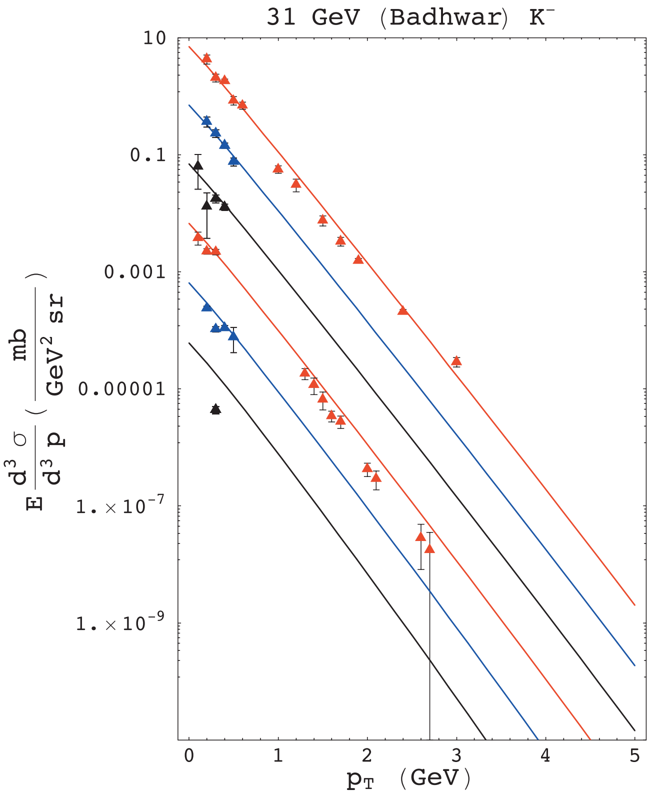

Figure 1. Badhwar parameterization vs. experiment (Alper et al. 1975) for inclusive K^- production in proton–proton collisions at $\sqrt{s} = 31$ GeV. The rapidity for the top curve is $y = 0.0$, and it increases in steps of 0.2 from the top to the bottom curves. The data and lines are multiplied successively by 0.1 to allow for a better separation.

(A color version of this figure is available in the online journal.)

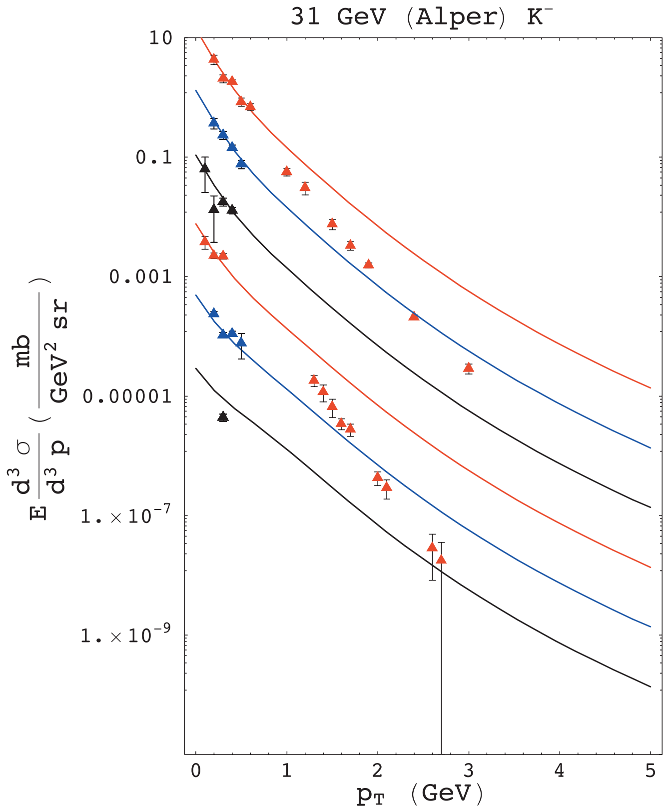

Figure 3. Same as Figure 1, except with Alper parameterization.
(A color version of this figure is available in the online journal.)

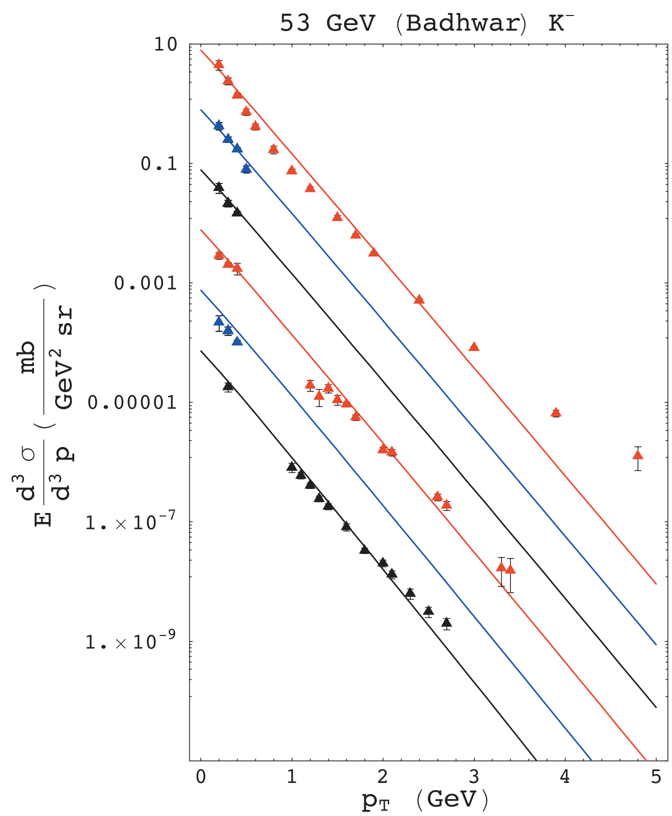

Figure 2. Same as Figure 1, except that $\sqrt{s} = 53$ GeV.
(A color version of this figure is available in the online journal.)

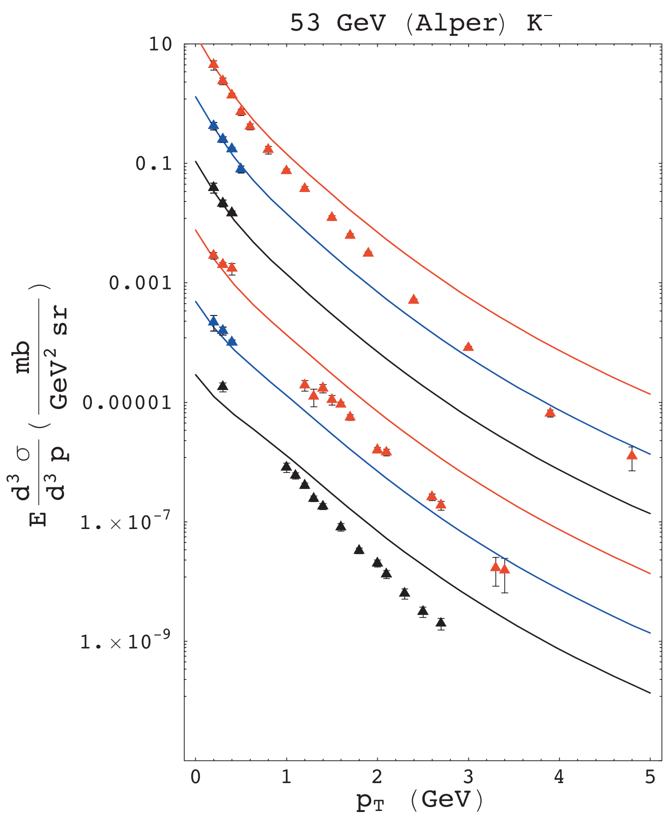

Figure 4. Same as Figure 2, except with Alper parameterization.
(A color version of this figure is available in the online journal.)

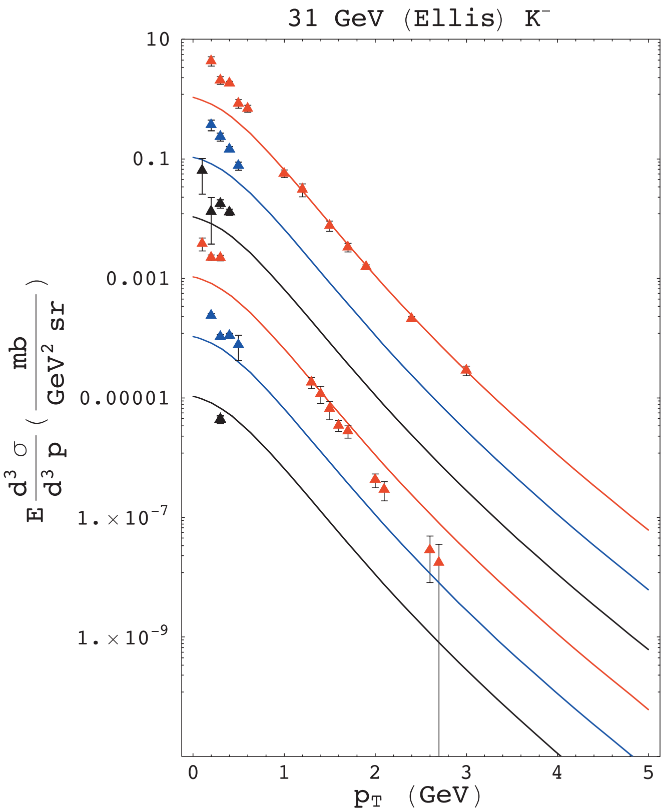

Figure 5. Same as Figure 1, except with Ellis parameterization. (A color version of this figure is available in the online journal.)

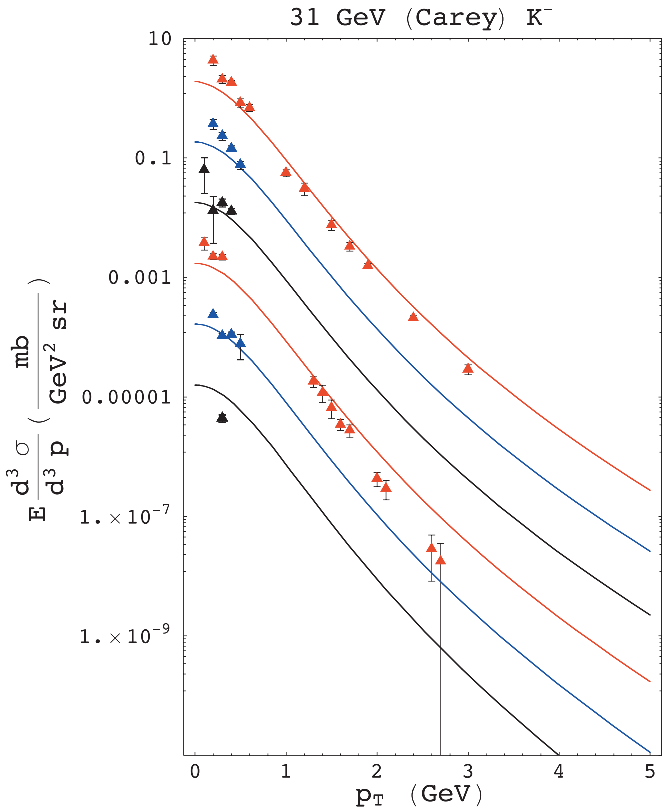

Figure 7. Same as Figure 1, except with Carey parameterization. (A color version of this figure is available in the online journal.)

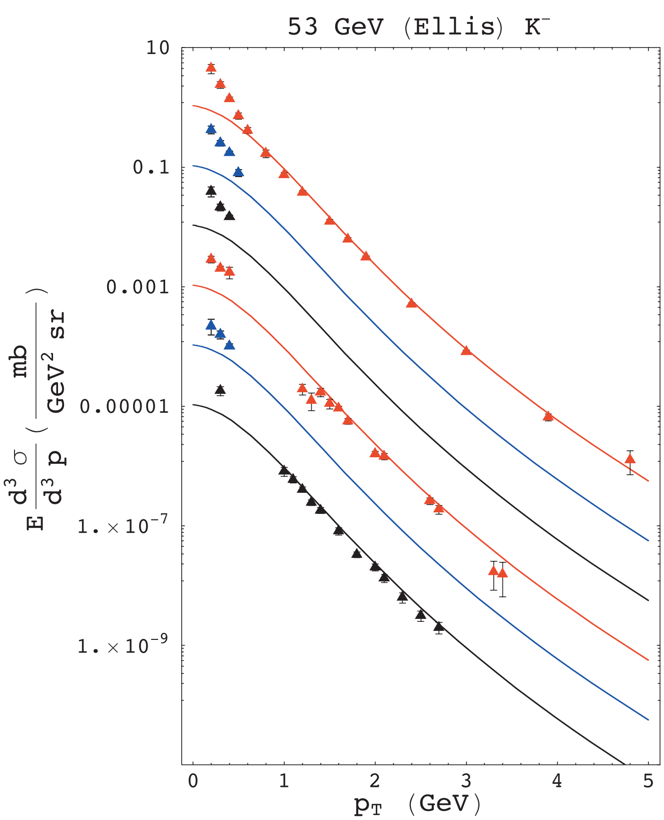

Figure 6. Same as Figure 2, except with Ellis parameterization. (A color version of this figure is available in the online journal.)

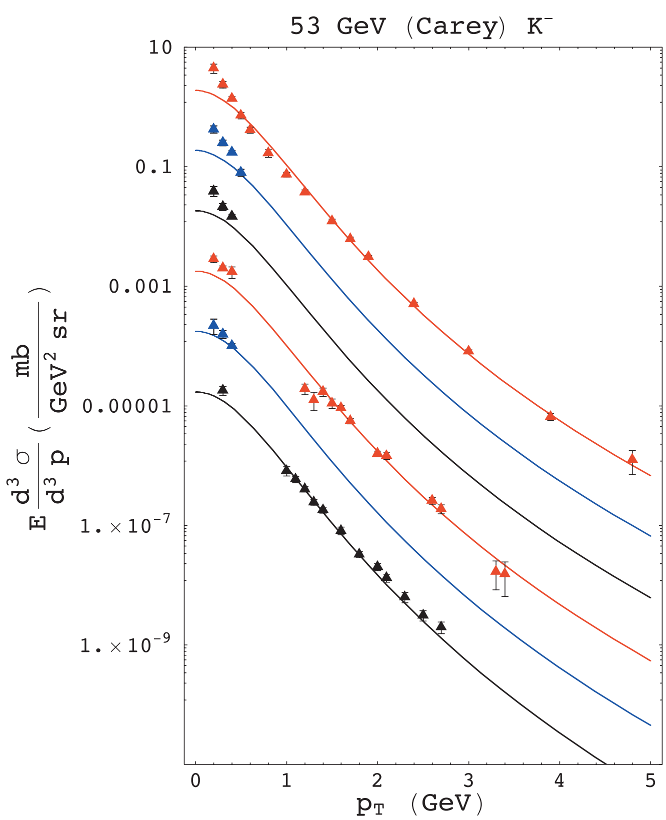

Figure 8. Same as Figure 2, except with Carey parameterization. (A color version of this figure is available in the online journal.)

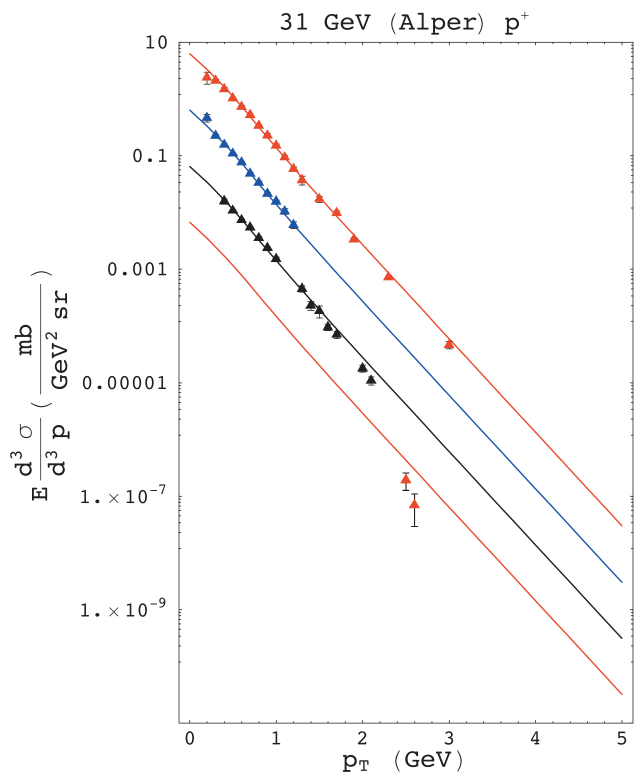

Figure 9. Alper parameterization vs. experiment (Alper et al. 1975) for inclusive proton production in proton–proton collisions at $\sqrt{s} = 31$ GeV. The rapidity for the top curve is $y = 0.0$, and it increases in steps of 0.2 from the top to the bottom curves. Data and lines are multiplied successively by 0.1 to allow for a better separation.

(A color version of this figure is available in the online journal.)

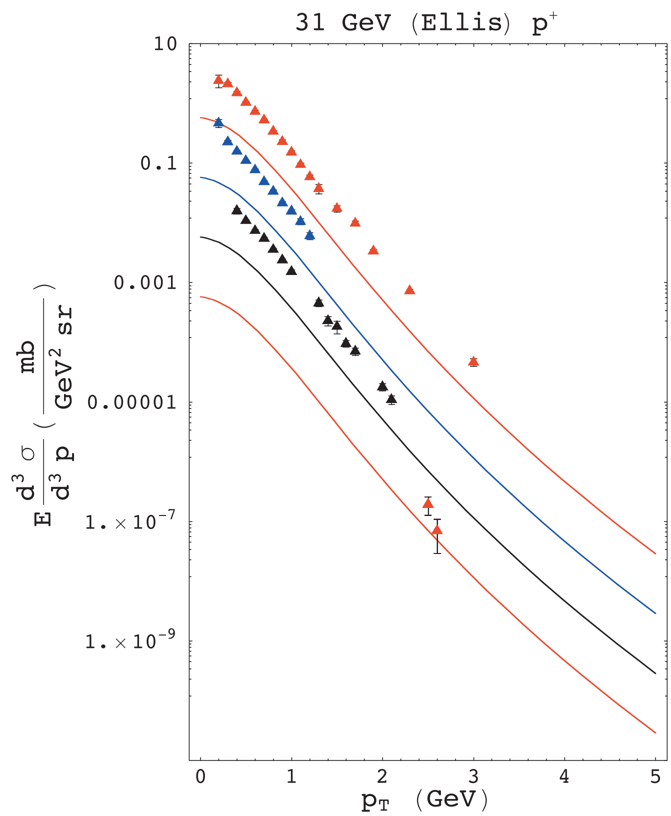

Figure 11. Same as Figure 9, except with Ellis parameterization.

(A color version of this figure is available in the online journal.)

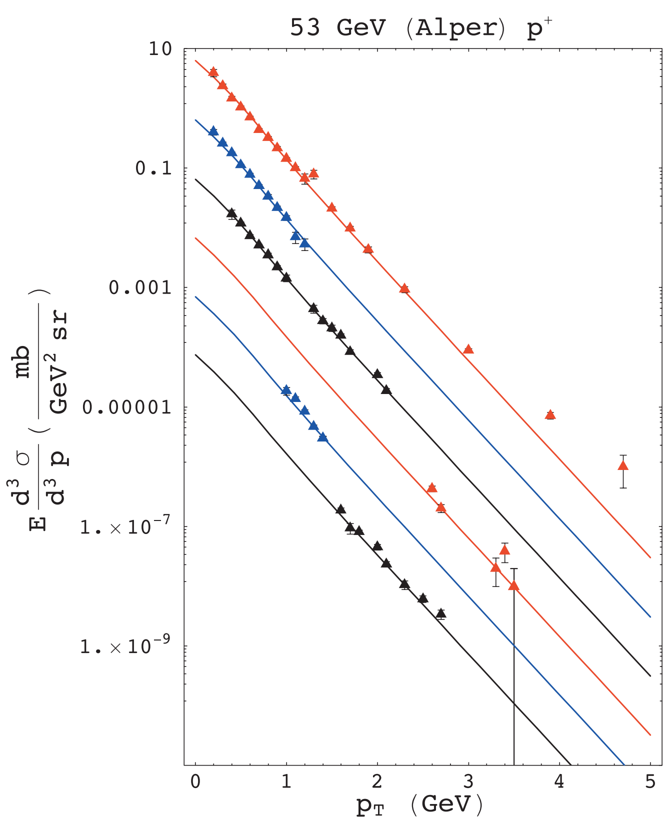

Figure 10. Same as Figure 9, except that $\sqrt{s} = 53$ GeV.

(A color version of this figure is available in the online journal.)

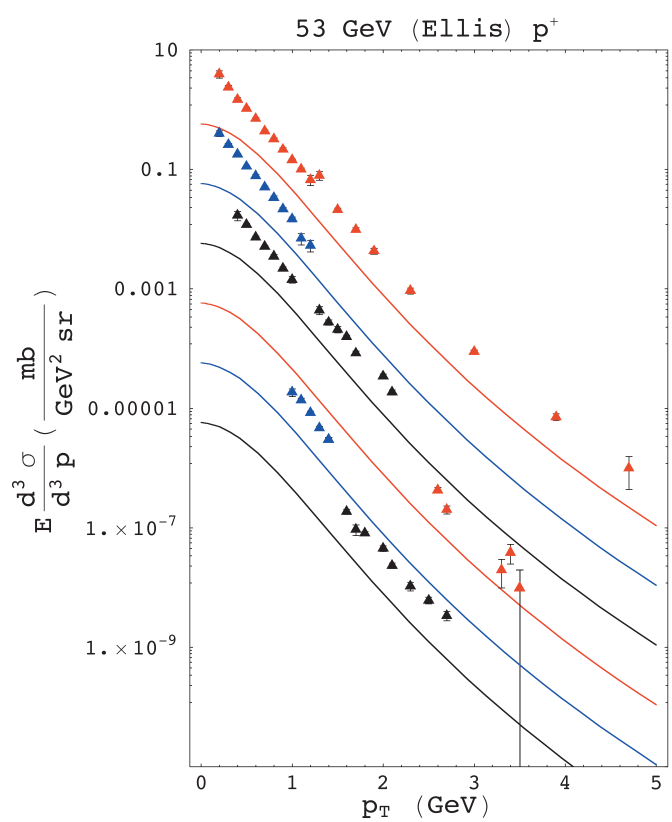

Figure 12. Same as Figure 10, except with Ellis parameterization.

(A color version of this figure is available in the online journal.)

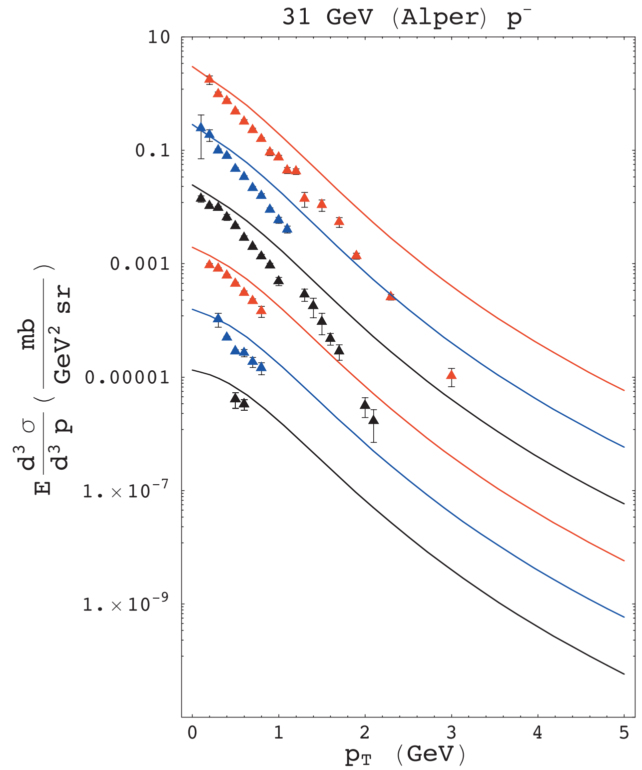

Figure 13. Alper parameterization vs. experiment (Alper et al. 1975) for inclusive antiproton production in proton–proton collisions at $\sqrt{s} = 31$ GeV. The rapidity for the top curve is $y = 0.0$, and it increases in steps of 0.2 from the top to the bottom curves. Data and lines are multiplied successively by 0.1 to allow for a better separation.

(A color version of this figure is available in the online journal.)

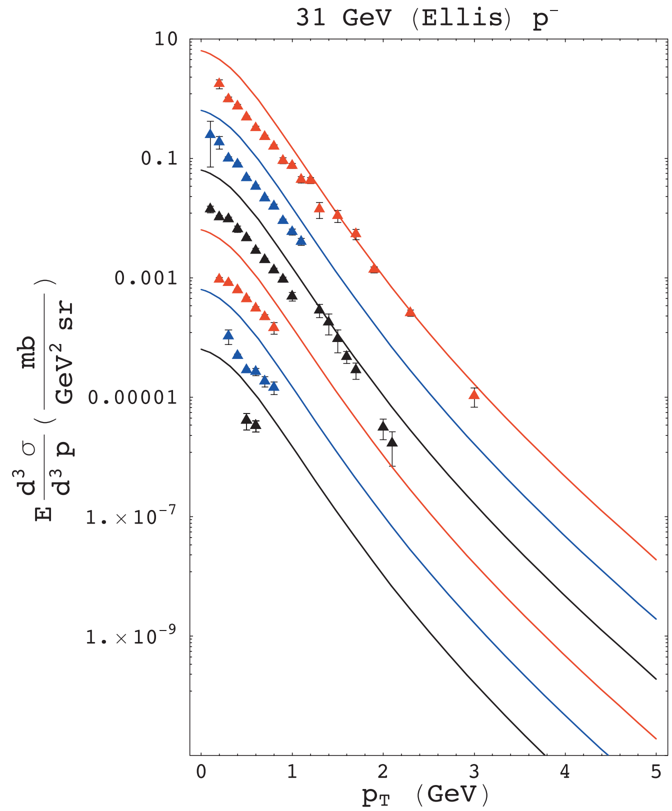

Figure 15. Same as Figure 13, except with Ellis parameterization.

(A color version of this figure is available in the online journal.)

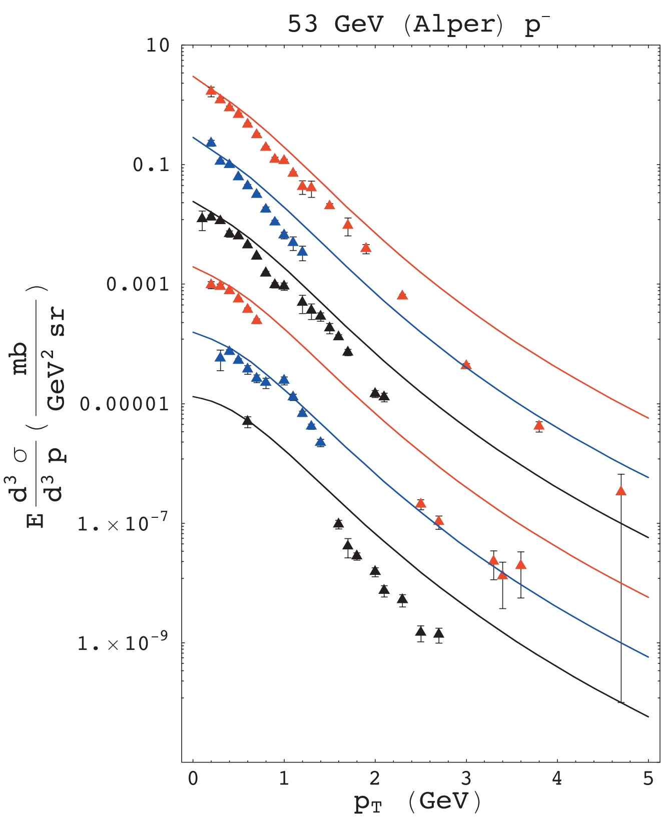

Figure 14. Same as Figure 13, except that $\sqrt{s} = 53$ GeV.

(A color version of this figure is available in the online journal.)

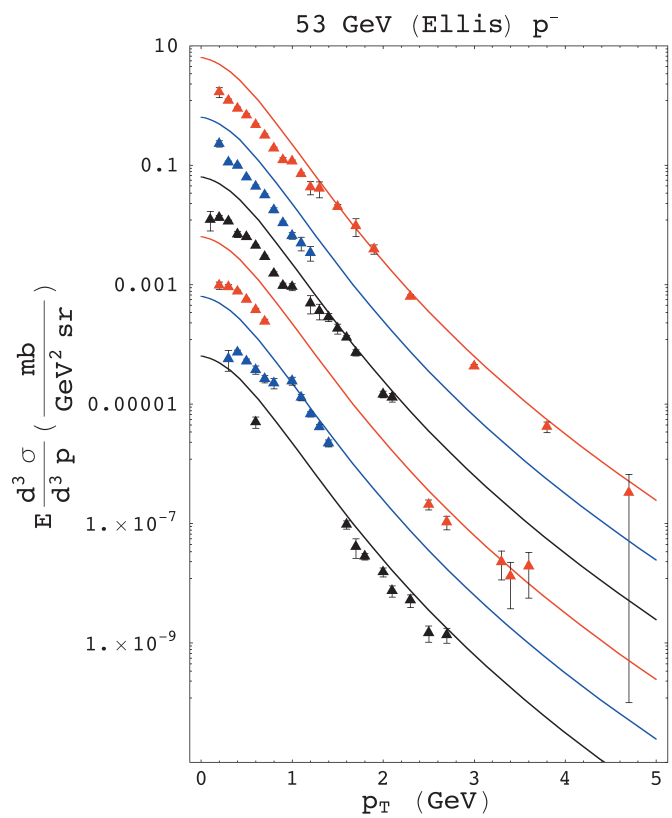

Figure 16. Same as Figure 14, except with Ellis parameterization.

(A color version of this figure is available in the online journal.)

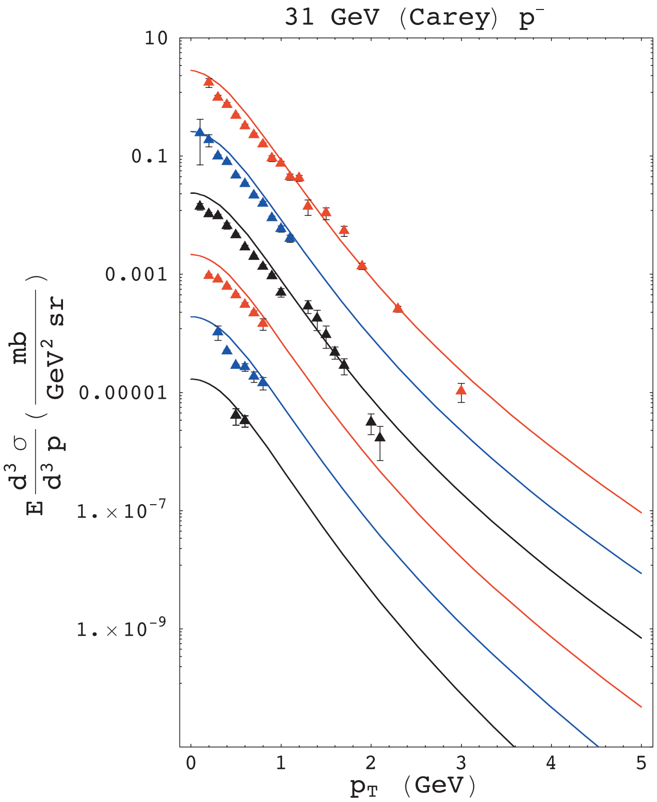

Figure 17. Same as Figure 13, except with Carey parameterization. (A color version of this figure is available in the online journal.)

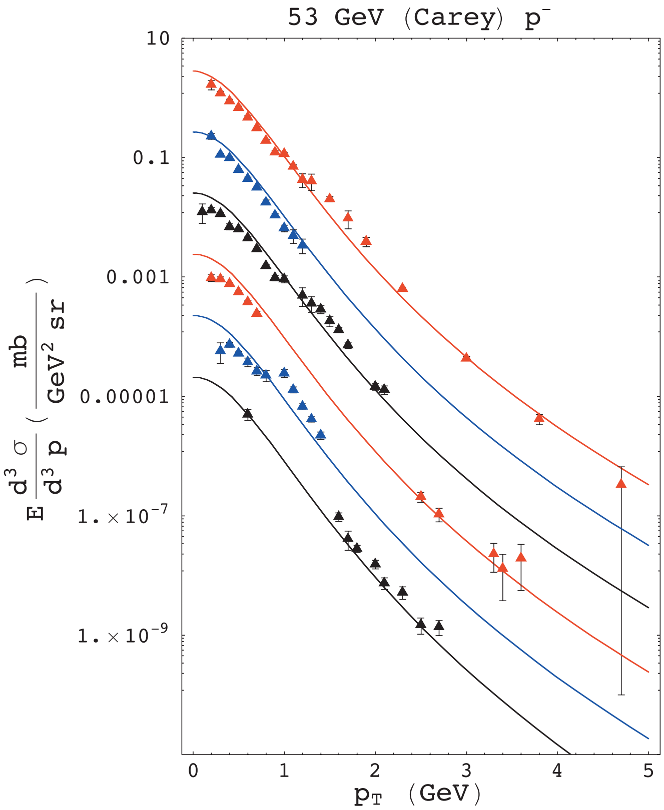

Figure 18. Same as Figure 14, except with Carey parameterization. (A color version of this figure is available in the online journal.)

curves. The data and lines are multiplied successively by 0.1 to allow for a better separation. The Badhwar and Alper parameterizations provide an excellent fit to data for low values of transverse momentum p_T, but fail for high p_T, with the Badhwar parameterization underpredicting data at high p_T and the Alper parameterization overpredicting at high p_T. The Ellis and Carey parameterizations work well at high p_T, but fail at low p_T. None of the parameterizations work well for all values of p_T.

Proton results are shown in Figures 9–12. The Badhwar parameterization is not available for protons. The Carey parameterization only applies to antiprotons. The Alper parameterization for protons is far superior to the Ellis parameterization. It is recommended that the Alper parameterization be used for protons.

Antiproton results are shown in Figures 13–18. The Badhwar parameterization is not available for antiprotons. The Alper and Ellis results for antiprotons are poor. The Carey results for antiprotons are quite good. It is recommended that the Carey parameterization be used for antiprotons.

4. CONCLUSIONS

Previous formulae for pion production in proton–proton collisions have been developed (Blattnig et al. 2000; Norbury & Townsend 2007). In the present work, inclusive production of kaons, protons, and antiprotons has been studied in proton–proton collisions for incident proton energies of $\sqrt{s} = 23$, 31, 45, 53, and 63 GeV. Various parameterizations have been compared to the experimental data of Alper et al. (1975). The Badhwar parameterization provides the best fit for charged kaon production. For proton production, the Alper parameterization is best, and for antiproton production the Carey parameterization works best. These formulae are suitable for inclusion in high-energy cosmic ray transport codes.

REFERENCES

Allison, J., et al. 2006, IEEE Trans. Nucl. Sci., 53, 270
Alper, B., et al. 1975, Nucl. Phys. B, 100, 237
Badhwar, G. D., Stephens, S. A., & Golden, R. L. 1977, Phys. Rev. D, 15, 820
Blattnig, S. R., Swaminathan, S., Kruger, A. T., Ngom, M., & Norbury, J. W. 2000, Phys. Rev. D, 62, 094030
Blume, C. 2007, Nucl. Phys. A, 783, 65
Carey, D. C., et al. 1974, Phys. Rev. Lett., 33, 330
d'Enterria, D. 2005, Eur. Phys. J. C, 43, 295
Domingo-Santamaria, E., & Torres, D. F. 2005, A&A, 444, 403
Ellis, S. D., & Stroynowski, R. 1977, Rev. Mod. Phys., 49, 753
Erlykin, A. D. 2007, Astropart. Phys., 27, 521
Fasso, A., et al. 2003, arXiv:hep-ph/0306267
Giller, M. 2008, J. Phys. G: Nucl. Part. Phys., 35, 023201
Huang, C. Y., Park, S. E., Pohl, M., & Daniels, C. D. 2007, Astropart. Phys., 27, 429
Kamae, T., Karlsson, N., Mizuno, T., Abe, T., & Koi, T. 2006, ApJ, 647, 692
Kaufman Bernado, M. M. 2005, arXiv:astro-ph/0504498
Kelner, S. R., Aharonian, F. A., & Bugayov, V. V. 2006, Phys. Rev. D, 74, 034018
Mahadevan, R., Narayan, R., & Krolik, J. 1997, ApJ, 486, 268
Moskalenko, I. V. 2007, Frascati Phys. Ser., 35, 115
Norbury, J. W., & Townsend, L. W. 2007, Phys. Rev. D, 75, 034001
Prodanovic, T., Fields, B. D., & Beacom, J. F. 2007, Astropart. Phys., 27, 10
Wilson, J. W., et al. 1991, Transport Methods and Interactions for Space Radiations, NASA Reference Publication 1257, available at http://ntrs.nasa.gov
Wilson, J. W., et al. 1995, HZETRN: Description of a Free-Space Ion and Nucleon Transport and Shielding Computer Program, NASA Technical Paper 3495, available at http://ntrs.nasa.gov