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Threshold meson production and cosmic ray transport

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Abstract

An interesting accident of nature is that the peak of the cosmic ray spectrum, for both protons and heavier nuclei, occurs near the pion production threshold. The Boltzmann transport equation contains a term which is the cosmic ray flux multiplied by the cross section. Therefore when considering pion and kaon production from proton–proton reactions, small cross sections at low energy can be as important as larger cross sections at higher energy. This is also true for subthreshold kaon production in nuclear collisions, but not for subthreshold pion production.

1. Introduction

The existence of the pion was predicted by Yukawa [1] in 1935 and discovered in the cosmic ray spectrum by Powell [2] in 1947. Seventy years later one would have thought that of all reactions, pion production in proton–proton collisions was thoroughly understood. There has been considerable interest recently in meson production from nucleon–nucleon collisions near the meson production threshold [3–6]. Pion production has been studied in the most detail. The reason these studies raised such interest was because the first calculations of total cross sections disagreed with experiment by a factor of 5 [5] and the differential cross sections for spin observables had the wrong shape, being convex instead of concave [7].

Particle reaction transport codes, such as GEANT [8] and FLUKA [9], are widely used in the design of accelerator experiments and the simulation of particle detectors. While typical cross sections away from threshold reach tens of millibarn, the cross sections near threshold are about a 1000 times smaller, typically in the microbarn region. The fact that theory and experiment disagreed so much in the threshold region was therefore of no real concern for particle simulation codes because the cross sections in the threshold region are so small.

Transport codes are also widely used in studies of cosmic rays. If one knows the cosmic ray spectrum incident on top of the Earth's atmosphere, then one can deduce the cosmic ray

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spectrum observed on the ground by transporting through the atmosphere. Similarly if one knows the cosmic ray spectrum incident on a spacecraft wall then one can deduce the radiation environment inside a spacecraft [10]. In cosmic ray research one turns this around to deduce the incident spectrum from the knowledge of the ground-based spectrum.

Transport codes are concerned with solving the Boltzmann transport equation. In the continuous slowing down and straight-ahead approximations, the one-dimensional Boltzmann equation can be expressed [10] as

$$\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial E} S_j(E) + \Sigma_j(E)\right]\phi_j(x, E) = \int \mathrm{d}E' \sum_k \Sigma_{jk}(E, E')\phi_k(x, E'), \tag{1}$$

where $S(E)$ is the stopping power, Σ is the macroscopic cross section and ϕ is the flux of particles at a given depth x and energy E. The macroscopic cross section is related to the microscopic cross section σ by $\Sigma = \rho\sigma$, where ρ is the target number density. Note that the flux ϕ is multiplied by the cross section σ. In a particle accelerator, the incident flux is normally a beam (constant flux) of particles with a well-defined energy. In cosmic ray physics the incident flux is represented by the incident cosmic ray spectrum, which is discussed in the review by Simpson [11]. In figure 5(a) of that review one can find the incident spectrum for protons and other nuclei. The peak of the spectrum occurs at an energy of about 300 MeV. The peak is caused by solar modulation of the interplanetary magnetic field and is known to move around somewhat depending on solar activity. Note that the peak of the spectrum occurs quite near the pion production threshold of 290 MeV. This is an interesting accident of nature. Above this energy the spectrum starts falling.

Now the Boltzmann equation contains flux multiplied by cross section. Therefore just because a cross section is small does not mean that it is unimportant. A small cross section might get multiplied by a large flux and a large cross section might get multiplied by a small flux, leading to approximately equal contributions from a transport point of view. This is possible when the incident flux is the cosmic ray spectrum at the top of the Earth's atmosphere.

2. Pion and kaon production in proton–proton collisions

Consider the exclusive reaction for production of neutral pions,

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + \pi^0. \tag{2}$$

Sample values of cosmic ray proton flux [11] and microscopic cross section are given in table 1, where the values are also multiplied together (we call this *importance*), as in the Boltzmann equation. It can be seen that the small cross sections near threshold can be of similar importance to larger cross sections away from threshold, the reason being due to the fact that the cosmic ray spectrum has a peak near threshold and falls steadily at higher energies. For example, the cross section at 375 MeV is 40 μb, while the cross section at 7 GeV is 1.7 mb, yet they give approximately equal contribution to the importance factor. Of course these do not compare to the importance of the cross sections at say 700 MeV and 1 GeV. However in a good transport code one will always include the cross section at a variety of energies and one would certainly go out to 7 GeV. The point is that if one does include such higher energy cross sections, then one may also need to include the cross sections near threshold as well because they are of comparable importance.

Similar statements can be made concerning kaon production as shown in table 2, where the lowest threshold reaction for producing kaons,

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{K}^+ + \Lambda + \mathrm{p} \tag{3}$$

Table 1. Neutral pion production from proton–proton collisions. (importance ≡ flux $\times \sigma$.) The numbers in square brackets after the cross sections indicate the reference number from where the cross section was taken. The peak in flux occurs near the pion threshold.

T (MeV)	Flux [11] $(\frac{1}{\mathrm{m^2\,sr\,s\,MeV}})$	σ (μb)	Reference	Importance $(\frac{\mu\mathrm{b}}{\mathrm{m^2\,sr\,s\,MeV}})$
290 ← Threshold				
325	2.0	7.7	[7]	15.4
350	2.0	17	[7]	34
375	1.8	40	[7]	72
400	1.8	86	[7]	155
700	1.3	2000	[12]	2600
1000	0.9	4000	[12]	3600
3000	0.24	3000	[12]	720
7000	0.06	1700	[12]	102
11500	0.01	1100	[12]	11

Table 2. Kaon production from pp $\to$ $\mathrm{K^+\Lambda p}$.

T (GeV)	Flux [11] $(\frac{1}{\mathrm{m^2\,sr\,s\,MeV}})$	σ (μb)	Reference	Importance $(\frac{\mu\mathrm{b}}{\mathrm{m^2\,sr\,s\,MeV}})$
1.58 ← Threshold				
1.60	0.6	0.16	[13]	0.1
1.82	0.5	7.4	[14]	3.7
1.90	0.4	8.6	[14]	3.4
2.06	0.3	16.5	[14]	5.0
2.85	0.2	50	[15]	10
4.83	0.1	50	[15]	5
7.29	0.06	50	[15]	3
30	0.002	16.6	[16]	0.03
50	0.0005	16.7	[16]	0.01

is considered. It can be seen that the importance of near-threshold reactions is comparable to reactions at all energies. Therefore it may be important to consider near-threshold production of kaons.

3. Subthreshold pion and kaon production in nucleus–nucleus collisions

Based on the above considerations, one might ask about the subthreshold pion production observed in nucleus–nucleus collisions some decades ago [17]. Here the word subthreshold refers to pions produced in nuclear collisions where the nucleus kinetic energy is below the nucleon–nucleon threshold. The typical way to express energies in nucleus–nucleus collisions is using A MeV. An energy of 290 A MeV would represent the nucleon–nucleon threshold. Simpson [11] also provides the cosmic ray spectrum for nuclei, such as He, C, Fe. Again the spectrum peak for these nuclei is right near the pion threshold. If the peak were below the pion threshold then subthreshold pion production in nuclear collisions would be important for the same reasons as given above. However because the nuclear peaks remain near the pion threshold then subthreshold pion production is not important in the sense described previously. This can be seen from table 3 where total cross sections for neutral pion production in carbon–carbon collisions below the pion threshold are presented. Due to a lack of total cross section

Table 3. π^0 production from C–C collisions. Note that the peak in flux occurs near the pion threshold.

T (AMeV)	Flux [11] $(\frac{1}{\mathrm{m^2\,sr\,s}\,A\,\mathrm{MeV}})$	σ (mb)	Reference	Importance $(\frac{\mathrm{mb}}{\mathrm{m^2\,sr\,s}\,A\,\mathrm{MeV}})$
60	5×10^{-3}	1.7×10^{-3}	[18, 19]	8.5×10^{-6}
74	6×10^{-3}	8.5×10^{-3}	[18, 19]	5.1×10^{-5}
84	7×10^{-3}	18.9×10^{-3}	[18, 19]	1.3×10^{-4}
290 ← Threshold				
325	7×10^{-3}	1		0.007
350	7×10^{-3}	2		0.01
375	6×10^{-3}	6		0.04
400	6×10^{-3}	12		0.07
700	4×10^{-3}	288		1.2
1000	3×10^{-3}	576		1.7
3000	5×10^{-4}	432		0.2
7000	1×10^{-4}	245		0.02
11500	2×10^{-5}	158		0.003

Table 4. K^+ production from Ni–Ni collisions. The approximate total experimental cross sections are from [20]. To obtain the flux for Ni, the flux of Fe was used but multiplied by 0.05, which is the average ratio of the abundance of Ni to Fe over all energy intervals [11].

T (AGeV)	Flux $(\frac{1}{\mathrm{m^2\,sr\,s}\,A\,\mathrm{MeV}})$	σ (mb)	Reference	Importance $(\frac{\mathrm{mb}}{\mathrm{m^2\,sr\,s}\,A\,\mathrm{MeV}})$
0.8	1.1×10^{-5}	0.9	[20]	1.0×10^{-5}
1.0	1×10^{-5}	2.7	[20]	2.7×10^{-5}
1.58 ← Threshold				
1.60	1×10^{-5}	0.5		5×10^{-6}
1.8	5×10^{-6}	57	[20]	2.9×10^{-4}
1.82	5×10^{-6}	25		1.3×10^{-4}
1.90	4.5×10^{-6}	29		1.3×10^{-4}
2.06	3.5×10^{-6}	56		2.0×10^{-4}
2.85	2.5×10^{-6}	168		4.2×10^{-4}
4.83	1×10^{-6}	168		1.7×10^{-4}
7.29	4×10^{-7}	168		6.7×10^{-5}
30	2.5×10^{-8}	56		1.4×10^{-6}
50	5×10^{-9}	56		2.8×10^{-7}

data, the experimental cross sections without reference numbers next to them were taken from table 1 and multiplied by 144, which is the product of the mass numbers of the projectile and target nuclei. Therefore they are approximate only. Again these cross sections are multiplied by the carbon cosmic ray flux and it can be seen that subthreshold cross sections are not important compared to cross sections at higher energies. Therefore it is probably safe to neglect subthreshold pion production in transport codes.

It is interesting to look at subthreshold kaon production because the kaon threshold is well above the pion threshold. Data are presented in table 4. Again due to a lack of total cross section data, the experimental cross sections without reference numbers next to them were taken from table 2 and multiplied by 3364, which is the product of the mass numbers of the projectile and target nuclei. It can be seen that subthreshold cross sections are as

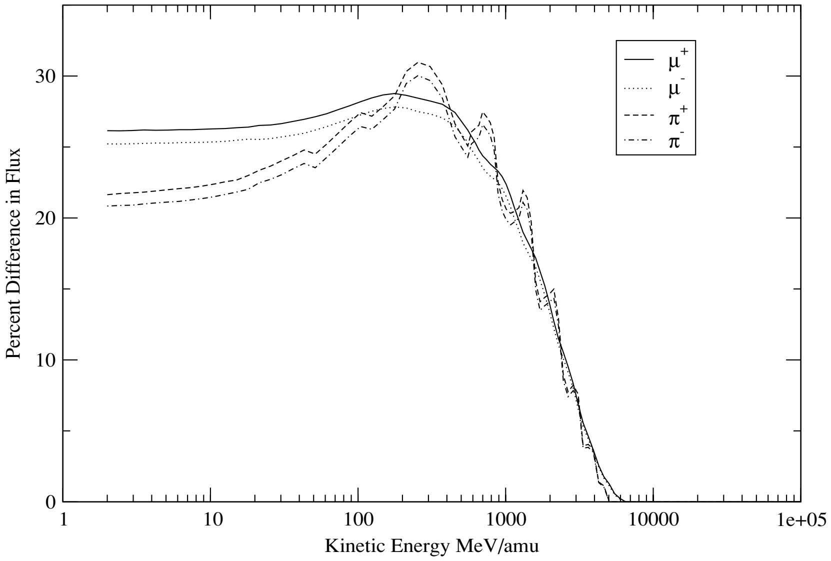

Figure 1. The per cent flux difference of charged muons and pions in the Mars atmosphere [24] at a depth of 38.9 g cm^{-2} from the 1977 GCR spectrum [23].

important as cross sections near 10 GeV. Therefore one can make the interesting conclusion that subthreshold kaon production may need to be included in transport codes. A better knowledge of the nucleus–nucleus total cross sections is necessary in order to make more definitive statements.

4. An example transport calculation

We have added this section on the advice of a referee, who wished us to do a sample transport calculation by artificially multiplying the cross section near threshold by a factor of 2 to see what the effects might be. The transport calculation was done with MESTRN [21], an extension of the NASA transport code HZETRN [22] that includes the production of charged pions and muons. One improvement that was made to the published version of MESTRN is the pion production spectral distributions from proton–proton collisions are now calculated by numerically integrating the Lorentz invariant differential cross sections of Badwar *et al* [23]. This was done to achieve better numerical convergence for low-density materials and to allow for transport through variable density materials.

The Martian atmospheric model of De Angelis *et al* [24] and the primary galactic cosmic ray (GCR) model of Badwar and O'Neill [25] were used as inputs for the transport calculation. Two runs of the transport code were performed. For one run, the cross section for the production of pions was artificially multiplied by a factor of 2 from threshold to a kinetic energy of 2 GeV.

Figure 1 shows the per cent flux difference of charged pions and muons after transport through 38.9 g cm^{-2} of Martian atmosphere corresponding to an approximate altitude of -10 km. A negative altitude indicates a valley or impact basin. The per cent flux difference is the flux of the MESTRN run with the cross section for pion production enhanced by a factor of 2 minus the nonenhanced flux. This value is then divided by the nonenhanced flux to give the per cent variation due to the enhancement of the pion production cross section near threshold.

The enhanced pion production cross sections led to an enhanced flux of charged pions and muons by approximately 20–30% in the kinetic energy region up to 1 GeV. The enhancement fades quickly after 1 GeV. We might expect this effect to be larger, but the produced pions decay quickly into muons which subsequently decay also. Muon decay in MESTRN is treated simply as a loss term. Subsequently, some of the muons that are produced from the increased pion production near threshold are not accounted for in these results.

5. Conclusions

Of course, particles with higher energy will be more penetrating than those of lower energy. This aspect of transport is not considered in the present paper. All that is considered is the product of the cross section with flux, which we called importance. We have shown that the small meson production cross sections near threshold can be just as important as larger cross sections at higher energy when transporting cosmic ray particles, because the shape of the cosmic ray spectrum enhances low energy cross sections, as used in the Boltzmann transport equation. In proton–proton reactions, this is true for pion production and especially true for kaon production. This leads us to speculate that near-threshold production of heavier hadrons may also need to be included in cosmic ray transport codes, because their thresholds are all above the pion threshold and the cosmic ray spectrum falls steadily as energy increases.

On the other hand this paper has shown that subthreshold pion production in nucleus–nucleus collisions is not important for cosmic ray transport because the energy of these reactions occurs below the peak of the nuclear cosmic ray spectra. However subthreshold kaon production probably may need to be included in transport codes and again we speculate that subthreshold production of heavier hadrons may need to be included.

It is important to note that the present paper does *not prove* that the reactions considered *must* be included in all cosmic ray transport codes. All we have done is to suggest that certain reactions with small cross sections at low energy have similar importance factors compared to larger cross sections at higher energy, and therefore, under certain circumstances, these *might* need to be included in transport codes. The circumstances will depend on the incident spectrum, the particles one is transporting, the medium through which transport is taking place and the particles of interest after transport through the medium.

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